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Research Article

The relationship between middle school students' mathematical literacy, geometry achievement, and Van Hiele geometric thinking levels

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This study investigates the relationships between middle school students' mathematical literacy, geometry achievement, and Van Hiele geometric thinking levels. Mathematical literacy was assessed in terms of students' ability to effectively use mathematics in real-life contexts, focusing on the processes of formulating, employing, and interpreting-evaluating. Conducted through a relational survey model and structural equation modeling, the study involved 550 eighth-grade students. Data were collected using the Geometry Achievement Test, the Van Hiele Geometric Thinking Test, and a Mathematical Literacy Test adapted from PISA items. Spearman correlation analysis revealed a very strong positive relationship between geometry achievement and mathematical literacy, and a strong positive relationship between geometric thinking levels and mathematical literacy. Structural equation modeling results indicated that geometry achievement had a substantial effect on mathematical literacy, particularly influencing the interpretation and evaluation process. Geometric thinking levels had a moderate effect on the formulation process and a lower effect on the application process. The model's fit indices demonstrated an excellent fit between the model and the data. Overall, the findings highlight the critical role of geometry achievement and Van Hiele-based geometric thinking in the development of mathematical literacy. In this context, geometry-focused instructional practices and activities that foster geometric thinking skills may significantly contribute to enhancing students' mathematical literacy.

Keywords: Geometry achievement, mathematical literacy, middle school students, structural equation modeling, Van Hiele geometric thinking level

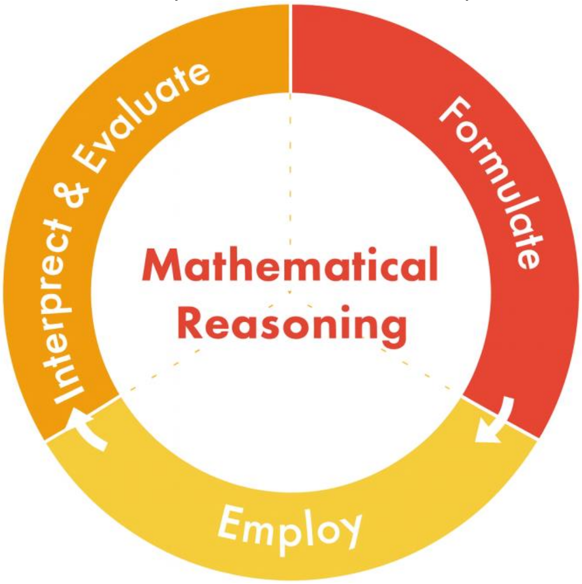
1. Introduction

Today, mathematics has become not only an academic discipline but also a skill that individuals need to use effectively in their daily lives, professional fields, and societal decision-making processes (Organisation for Economic Co-operation and Development [OECD], 2018). In this context, the concept of mathematical literacy, defined as the ability to apply mathematical content (concepts, facts, procedures, and tools) in real-world situations, has gained increasing importance (Stacey & Turner, 2014). Mathematical literacy refers to individuals' capacity to understand mathematical concepts, apply mathematical knowledge in various contexts, and use it effectively during problem-solving processes (OECD, 2022). According to McCrone and Dossey (2007), mathematical literacy involves recognizing and interpreting the functions of mathematics in daily life, and using it to meet individual needs through a capacity to engage with mathematical ideas. Individuals with strong mathematical literacy are said to possess skills such as prediction, data interpretation, analysis, and reasoning-based communication (Ojose, 2011). Therefore, mathematical literacy is considered as essential as basic reading and writing skills.

In order to be mathematically literate, individuals must possess knowledge not only of arithmetic, but also of key areas of mathematics such as measurement, geometry, statistics, probability, and algebra (National Council of Teachers of Mathematics [NCTM], 1989). Consequently, assessing and evaluating individuals' level of mathematical literacy is crucial for countries to analyze the effectiveness of their educational policies. In this regard, the Programme for International Student Assessment [PISA], organized by the OECD, enables international comparisons by measuring students' mathematical literacy levels. The PISA assessment framework approaches mathematical literacy through three dimensions: (a) mathematical content, (b) context, and (c) mathematical processes (OECD, 2022).

Mathematical processes refer to how individuals use mathematics in understanding problems, developing solutions, and evaluating outcomes. According to the PISA framework, these processes fall into three core skill areas: formulating, which involves translating real-world problems into mathematical form; employing, which refers to developing solutions using appropriate mathematical methods; and interpreting and evaluating, which involves reflecting on the results within the problem context (OECD, 2022) (see Figure 1).

Figure 1
The relationship between mathematical processes and reasoning in solving real-life problems



According to the PISA assessment framework, mathematical content is categorized under four main areas: quantity, uncertainty and data, change and relationships, and space and shape. The "space and shape" domain encompasses students' abilities to recognize geometric concepts, perceive spatial relationships, and apply these skills to real-life problem contexts. This category also includes sub-skills such as position, direction, symmetry, transformation, and mental representation of three-dimensional objects (OECD, 2022).

Geometry achievement plays a crucial role in developing students' analytical thinking, spatial reasoning, and problem-solving skills, beyond simply recognizing shapes or performing calculations (Sherard, 1981). The ability to solve space and shape-related questions in PISA is directly linked to students' understanding and application of geometric concepts (OECD, 2022). Therefore, geometry achievement is not only an indicator of academic success but also a significant predictor of students' levels of mathematical literacy. As mathematical literacy involves modeling, interpreting, and evaluating real-world problems using mathematical tools, it is strongly associated with domains like geometry that require concrete and visual thinking (Stacey & Turner, 2014).

The Van Hiele Model of Geometric Thinking provides a structured explanation of how students develop geometric understanding and how these thinking levels influence mathematical literacy. At the lower levels of the model, students recognize shapes based on their visual features, while those at higher levels analyze relationships between shapes, make abstractions, and draw logical inferences (Usiskin, 1982; Van Hiele, 1986). Studies have shown that students who advance through Van Hiele levels perform better on space and shape-related mathematical literacy tasks in PISA (Muliawanti & Kusuma, 2021; Nurhayati & Lutfianto, 2020). Thus, enhancing students' thinking levels is as important as developing content knowledge to improve mathematical literacy.

This study focuses on students' geometric thinking levels and geometry achievement within the space and shape dimension of mathematical literacy. Geometric thinking is a vital component of mathematical literacy, influencing how individuals perceive, analyze, and model the physical world around them (Clements & Battista, 1992). Therefore, the quality of geometry education is considered a key factor in shaping students' success and geometric reasoning within the space and shape domain.

Van Hiele (1986) proposed a five-level developmental model that students typically follow in learning geometry: visualization, analysis, informal deduction, formal deduction, and rigor (Usiskin, 1982). Research highlights that students progress through these stages at different rates and that instructional methods should be adapted accordingly (Battista, 2007). The Van Hiele model remains a valid theoretical framework for understanding middle school students' geometric thinking. Recent studies have confirmed its relationship with geometry performance, reasoning, and proof skills. For example, Çetin (2022) and Başarır (2024) found that students' Van Hiele levels closely relate to their proportional and spatial reasoning skills. Hatip (2022) showed that proof-writing and justification performance varied according to students' Van Hiele levels. Similarly, Akıl (2020) and Demir (2019) reported that students' achievements in specific geometry topics (e.g., transformation geometry, circles) aligned with their geometric thinking levels. Research by Çulhan and Çontay (2023) also revealed variations in students' geometric discourse across levels, while Türnüklü and Özcan (2014) demonstrated parallels between students' knowledge construction processes and their Van Hiele levels. These findings suggest that using the model in education can help students learn geometry concepts more consciously and meaningfully. Supporting geometry education at an early age plays a critical role in developing advanced mathematical thinking skills, making students' Van Hiele levels important in fostering mathematical literacy.

This study specifically examines the "space and shape" content domain, which is directly related to geometry education and includes essential skills such as perceiving spatial relationships, recognizing geometric shapes, and visual-spatial reasoning. International assessments have reported relatively lower student performance in this area compared to other domains. PISA reports emphasize that students underperform in the space and shape domain and that more emphasis should be placed on improving this area (OECD, 2022). Therefore, this research aims to explore the space and shape dimension of mathematical literacy in depth.

Trapsilasiwi et al. (2019) emphasized the critical role of geometry and geometric reasoning in solving space and shape-related problems. Similarly, Nurhayati and Lutfianto (2020) noted that students utilize various mathematical thinking skills while solving such problems, including visualization, analysis, generalization, and validation. Nurutami et al. (2019) found that most students struggled to accurately analyze geometric relationships and failed to make appropriate mathematical inferences from contextual situations, indicating a low level of success in this domain. These findings support the idea that greater emphasis should be placed on geometric thinking to improve students' mathematical literacy.

This study aims to examine in detail the influence of geometry achievement and geometric thinking levels on students' mathematical literacy in the space and shape domain and its sub-processes. It holistically investigates the interactions among mathematical literacy, geometry achievement, and Van Hiele geometric thinking levels. Within the PISA framework, the space and shape domain assesses students' abilities to apply geometric thinking to real-life contexts. Geometry achievement and geometric thinking levels are considered fundamental cognitive components shaping mathematical literacy.

Some researchers highlighted that the Van Hiele model supports not only geometry achievement but also creative and relational thinking skills (e.g. Breyfogle & Lynch, 2010; Erdoğan et al., 2009). Similarly, Van Hiele geometric thinking levels have been associated with students' comprehension and reasoning abilities in geometry (Başarır, 2024; Çetin, 2022). However, previous studies have not directly centered on the concept of mathematical literacy but have instead focused on related sub-skills. In the literature, no comprehensive study examines the relationship between the space and shape dimension of mathematical literacy and either Van Hiele levels or geometry achievement. Therefore, this study seeks to fill this theoretical gap by systematically investigating the interrelationships among the three variables. Additionally, its unique contribution lies in analyzing mathematical literacy not only as a general competency but also through its three functional processes: formulating, employing, and interpreting-evaluating.

The findings are expected to inform the development of instructional strategies that address both students' cognitive levels in geometry and the processes of mathematical literacy. Moreover, the study is anticipated to provide valuable insights for the design of mathematics curricula. In this sense, it contributes both to the theoretical knowledge base and to practical teaching applications. Accordingly, the aim of this study is to reveal the relationship between middle school students' mathematical literacy, Van Hiele geometric thinking levels, and geometry achievement. The research question is as follows: Is there a significant relationship between middle school students' mathematical literacy and its sub-processes and their Van Hiele geometric thinking levels and geometry achievement, and do these variables significantly predict one another? Based on this, the sub-questions of the study are:

RQ 1) Is there a relationship between middle school students' levels of mathematical literacy and its sub-processes and their geometry achievement and Van Hiele geometric thinking levels?

RQ 2) Do geometry achievement and Van Hiele geometric thinking levels significantly affect students' mathematical literacy and its sub-processes?

2. Method

In this study, a relational survey model and structural equation modeling [SEM] were used to determine the relationships among variables and to uncover causal connections. The relational survey model was utilized to identify and quantify the relationships among variables such as geometry achievement, geometric thinking levels, mathematical literacy levels, and the sub-processes of formulating, employing, interpreting and evaluating. However, since the relational survey model alone cannot reveal causal relationships (Creswell, 2014), SEM was employed to examine the causal effects in detail. SEM was chosen because it allows the simultaneous analysis of multiple dependent and independent variables (Kline, 2011), enabling the researchers to test and interpret complex relationships among variables within the study.

2.1. Participants

The sample of the study consisted of 550 eighth-grade students attending schools affiliated with the Ministry of National Education in the province of Iğdır. A stratified sampling method was used to select the sample. Stratified sampling involves dividing the population into subgroups (strata) based on specific characteristics and then selecting participants from each subgroup either randomly or systematically (Teddlie & Yu, 2007), with the aim of better representing the overall structure of the population (Creswell & Creswell, 2017). Accordingly, schools were categorized into low, medium, and high academic achievement groups based on national and local standardized exam results (e.g., High school entracnce exam scores), and participants were randomly selected from these schools.

2.2. Data Collection Tools and Procedure

The data collection tools used in this study were the Geometry Achievement Test, the Van Hiele Geometric Thinking Test, and the Mathematical Literacy Test. To assess students' geometry achievement levels, a Geometry Achievement Test was developed. The test development process followed standard procedures including defining the purpose, constructing a table of specifications, writing items, seeking expert review, conducting a pilot study, performing item analysis, finalizing the test, and evaluating validity and reliability (Crocker & Algina, 1986).

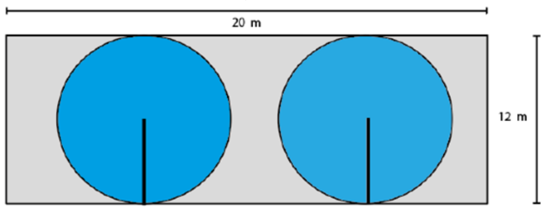
After defining the test's purpose, a table of specifications was created to ensure the alignment of test items with the learning outcomes and to establish content validity. Based on this table, items were written in accordance with relevant learning outcomes and were reviewed by subject matter experts. Revisions were made based on expert feedback, followed by a pilot implementation. The difficulty and discrimination indices of items were calculated and further adjustments were made accordingly. Item analysis showed that the discrimination indices of the Geometry Achievement Test ranged between 0.30 and 0.53, indicating that most items had moderate to high discriminating power. The difficulty indices ranged from 0.35 to 0.63, suggesting that the test had a moderate difficulty level. The internal consistency of the test was calculated using the KR-21 formula, resulting in a reliability coefficient of 0.82. This value indicates a high level of reliability, as a KR-21 value of 0.80 or above suggests that the test produces consistent and reliable results (Büyüköztürk, 2021). Table 1 presents the learning outcomes targeted by the 15 multiple-choice questions in the Geometry Achievement Test.

Table 1
Learning Outcomes Addressed by the Geometry Achievement Test Questions

Question No	*Learning Outcome*
1, 2	Identifies and examines the properties of corresponding, alternate interior, alternate exterior, and vertical angles formed by a transversal with two parallel lines; determines which angles are equal or supplementary; solves related problems.
3, 4, 5	Determines the diagonals, interior and exterior angles of polygons; calculates the sum of interior and exterior angle measures.
6, 7, 13, 14	Solves problems related to area.
8	Determines the measures of central angles in a circle, the arcs they intercept, and the relationships among these measures.
9, 10	Calculates the area of a circle and a sector.
11, 15	Calculates the circumference of a circle and the length of an arc.
12	Draws 2D representations of 3D objects from different perspectives.

Question 10, developed within the scope of the learning outcome "Calculates the area of a circle and a sector," is presented in Figure 2.

Figure 2
Geometry Achievement Test – Question 10



Two identical circular pools will be constructed on a rectangular ground as shown in the figure, with their edges tangent to the shorter side of the ground. The model presents the length and width of the rectangular area. Accordingly, what is the area (in m^2) of the region **where no pool will be built**? (Use $\pi = 3$)

A) 132 B) 66 C) 48 D) 24

In this question, a scenario is presented in which two identical circular pools are placed on a rectangular surface, and students are asked to calculate the area of the region where no pool will be built. In this context, students are expected to recall and apply the area formulas for rectangles and circles, interpret the positioning and arrangement of the shapes, and develop meaningful solution strategies—demonstrating a range of cognitive and geometric skills. The question corresponding to the learning outcome "Draws 2D representations of 3D objects from different perspectives" is presented in Figure 3.

Figure 3
Geometry Achievement Test – Question 12

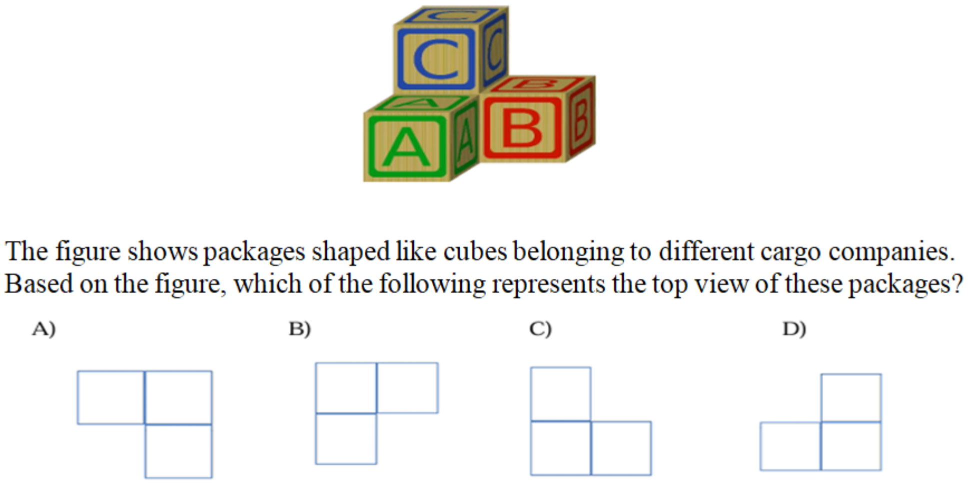


Figure 3 presents Question 12, which aims to assess students' visual-spatial thinking skills. In this question, students are asked to determine the top view of a structure composed of three cubes labeled A, B, and C. This item is designed to measure students' abilities to mentally rotate objects, visualize from different angles, and analyze spatial relationships.

To determine students' geometric thinking levels, the Van Hiele Geometric Thinking Test was used. This test, developed by Usiskin (1982), was translated into Turkish and validated by Duatepe (2000). The Cronbach's alpha reliability coefficients calculated by Duatepe ranged from 0.59 to 0.82. The test consists of five levels, with five questions per level, totaling 25 questions. To be considered successful at a particular level, a student must answer at least three out of five questions correctly at that level (Usiskin, 1982).

To assess middle school students' mathematical literacy levels, a Mathematical Literacy Test was developed. This test was structured to reflect the mathematical processes and was adapted from PISA pilot and main study questions from previous years (see Table 2).

Table 2
Questions and Processes in the Mathematical Literacy Test

Mathematical Process	*Level*	*Points*	*Question*
Employing	2	4	Stairs from PISA 2003 (OECD, 2005)
Employing	5	10	Oil Spill from PISA 2012 (OECD, 2013)
Interpreting & Evaluating	4	8	Seeing the Tower from PISA 2006 (OECD, 2007)
Formulating	3	6	Buying an Apartment from PISA 2012 (OECD, 2013)
Interpreting & Evaluating	1	2	Garage from PISA 2012 (OECD, 2013)
Formulating	6	12	Revolving Door 2 from PISA 2012 (OECD, 2013)

The Mathematical Literacy Test includes items related to the content area of space and shape. Care was taken to include questions representing all six levels of mathematical literacy and to balance the sub-processes of (a) formulating, (b) employing, and (c) interpreting & evaluating. Among these, "Oil Spill" (PISA, 2012) was a pilot question, while "Seeing the Tower" (PISA, 2006), "Buying an Apartment" (PISA, 2012), "Garage" (PISA, 2012), "Stairs" (PISA, 2003), and "Revolving Door 2" (PISA, 2012) were all used in main PISA assessments.

2.3. Data Analysis

Before proceeding with the analysis of the quantitative data obtained in the study, the normality of the variables was examined to determine the applicability of parametric tests. In this regard, the Shapiro-Wilk test, commonly used for small and medium-sized samples, was employed to assess the assumption of normal distribution. The test results are presented in Table 3.

Table 3
Shapiro-Wilk Test Results for Variables

Variable	*Statistic*	*p*
Geometry Achievement	0.98	$p < .05$
Van Hiele Scores	0.73	$p < .05$
Mathematical Literacy Scores	0.82	$p < .05$
Formulating	0.43	$p < .05$
Employing	0.74	$p < .05$
Interpreting and Evaluating	0.16	$p < .05$
Formulating z-score	0.43	$p < .05$
Employing z-score	0.74	$p < .05$
Interpreting and Evaluating z-score	0.16	$p < .05$

As shown in Table 3, the geometry achievement scores and the scores for the sub-processes of mathematical literacy—(a) formulating, (b) employing, and (c) interpreting and evaluating—do not follow a normal distribution. Additionally, because the maximum possible scores for the sub-processes of mathematical literacy vary, these scores were standardized into z-scores. This standardization enabled the comparison of processes with different score distributions. The relevant statistical analyses were conducted using these standardized z-scores. However, it should be noted that z-scores reflect individuals' relative positions within a group rather than their absolute performance; thus, the results were interpreted accordingly.

In cases where normal distribution is not achieved and ordinal variables are involved, the Spearman correlation test provides valid and reliable results for calculating the correlation between two variables (Field, 2024). Accordingly, the Spearman correlation test was used to determine the relationships among variables, and a correlation matrix was created by calculating the correlation coefficients between each pair of variables.

To determine the effects of independent variables on dependent variables, structural equation modeling was used. SEM analyses were conducted using the RStudio software. SEM is an effective method for analyzing relationships among variables (Kline, 2011). The analysis process consisted of defining the theoretical framework of the model, constructing it visually, estimating parameters, evaluating model fit, and interpreting the results.

In this study, geometry achievement and Van Hiele geometric thinking levels were identified as independent variables, and their effects on mathematical literacy and its sub-processes were examined. After the model was constructed, theoretical relationships were visualized using a path diagram. Once the theoretical model was specified, parameter estimates were calculated. At this stage, path coefficients (β), which indicate the extent to which dependent variables are explained by independent variables, were computed.

In estimating model parameters, the Diagonally Weighted Least Squares [DWLS] method was preferred, considering the distribution of the data and the nature of the variables. DWLS is particularly effective for analyzing non-normally distributed and categorical data (Li, 2016). For analyses involving non-normally distributed and ordinal variables, DWLS provides lower error rates and more consistent, reliable results (Flora & Curran, 2004).

After estimating the model parameters, the fit of the theoretical model to the observed data was evaluated. For this evaluation, indices such as the Comparative Fit Index [CFI] and the Tucker-Lewis Index [TLI] are used in structural equation models to determine the adequacy of model fit (Hu & Bentler, 1999).

3. Results

The first sub-problem of the study is stated as: "Is there a relationship between middle school students' levels of mathematical literacy and its sub-processes, their geometry achievement levels, and Van Hiele geometric thinking levels?" In this context, the correlation matrix illustrating the relationships between middle school students' levels of mathematical literacy and its sub-processes, their geometry achievement levels, and Van Hiele geometric thinking levels is presented in Table 4.

Table 4
Correlation Matrix among Variables

Variable	*GTL*	*GA*	*MLL*	*F*	*E*	*IE*
GTL	1.00	–	–	–	–	–
GA	.68***	1.00	–	–	–	–
MLL	.70***	.90***	1.00	–	–	–
F	.69***	.79***	.86***	1.00	–	–
E	.45***	.58***	.68***	.40***	1.00	–
IE	.31***	.63***	.72***	.44***	.36***	1.00

Note. ***: $p < .01$; GA: Geometry Achievement; GTL: Geometric Thinking Level; MLL: Mathematical Literacy Level; F: Formulating; E: Employing; IE: Interpreting and Evaluating; The *z*-scores were taken into account when calculating the scores for Formulating, Employing, and Interpreting and Evaluating.

As shown in Table 4, all correlations among the variables are statistically significant and positive at the $p < .01$ level. The very high correlation observed between GTL (Geometric Thinking Level) and MLL (Mathematical Literacy Level) ($r = .70$) indicates that individuals with higher geometric thinking levels also tend to have higher levels of mathematical literacy. Among the sub-processes, the strongest relationship with GTL is observed with F (Formulating) ($r = .69$). Additionally, GTL has a moderate positive relationship with E (Employing) ($r = .45$), and a relatively weaker but still moderate relationship with IE (Interpreting and Evaluating) ($r = .31$).

The strong correlation between GA (Geometry Achievement) and MLL ($r = .90$) reveals a powerful relationship between geometry achievement and mathematical literacy level. This finding suggests that individuals with high geometry achievement also tend to have high mathematical literacy. When examining the relationship between GA and the sub-processes of mathematical literacy, the highest correlation is with F ($r = .79$). GA also shows strong and positive correlations with E ($r = .58$) and IE ($r = .63$).

When examining the relationships between MLL and its sub-processes, the strongest correlation is observed between MLL and F ($r = .86$).

The second sub-problem of the study is stated as: "Do middle school students' geometry achievement level and Van Hiele geometric thinking levels statistically significantly affect mathematical literacy and its sub-processes?" To examine the causal structure among the variables in more depth, a structural equation model was employed. Within this scope, a structural model was established to determine whether geometry achievement level and Van Hiele geometric thinking levels statistically significantly affect mathematical literacy levels and the processes of formulating, interpreting and evaluating, and employing. The presumed relationships among the variables in the model are visualized through a path diagram. The path diagram for the model established in the study is presented in Figure 4.

In Figure 4, it is observed that GA has statistically significant and positive effects on MLL and its sub-processes: F, IE, and E. The GTL variable also shows significant and positive effects on MLL and the sub-processes F and E. However, GTL does not have a statistically significant effect on IE ($p > .01$). Therefore, the relationship between GTL and IE was removed from the model, and the revised path diagram along with the corresponding coefficients is presented in Figure 5.

Figure 4
Path Diagram of the Relationships among Variables

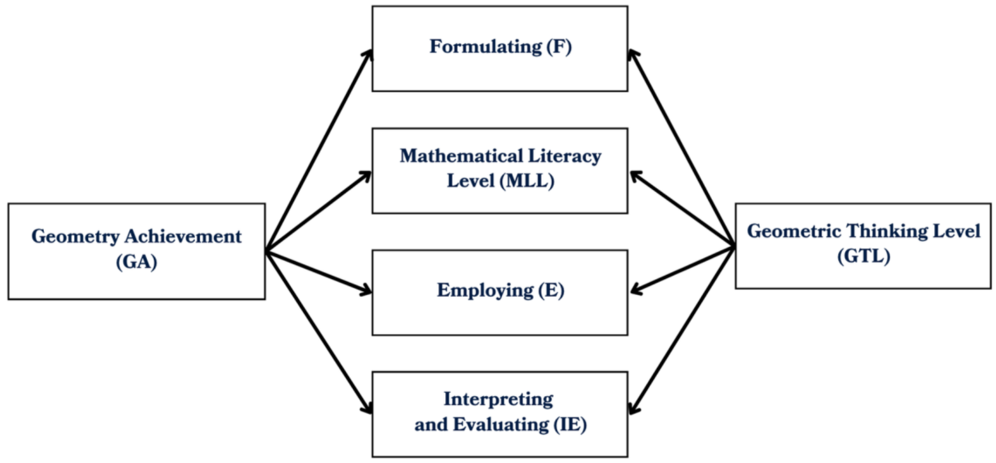
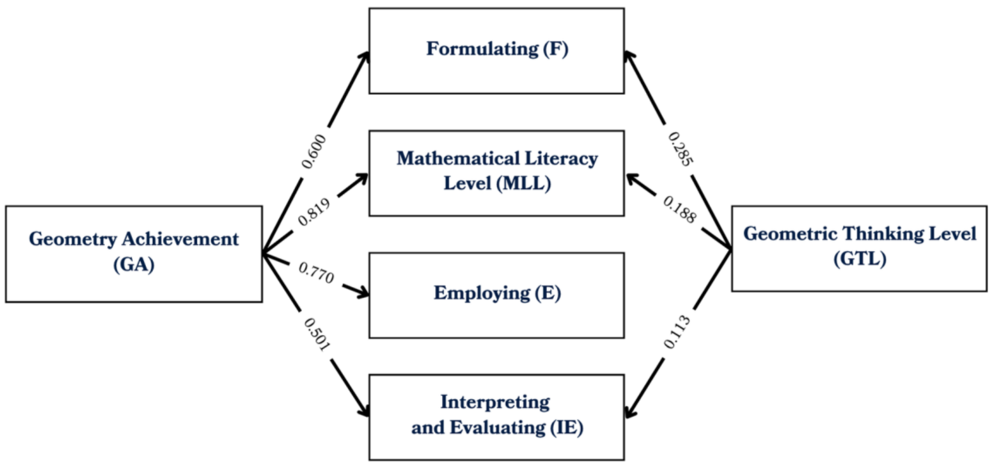


Figure 5
Revised Path Diagram



Upon examining Figure 5, it is observed that in the revised model, the variable GA continues to have significant and positive effects on MLL and its sub-processes: F (Formulating), E, and IE. The variable GTL also has positive effects on MLL and the sub-processes F and E. The regression coefficients corresponding to these relationships are presented in Table 5.

Table 5
Regression Coefficients (Path Coefficients) for Variables

Independent Variable	*Dependent Variable*	*SE*	*z*	*p*	*Standardized Coefficient*
GTL	F	0.039	10.158	<.01	0.285
GA	F	0.002	19.703	<.01	0.600
GTL	E	0.063	2.464	<.01	0.113
GA	E	0.003	12.351	<.01	0.501
GA	IE	0.003	17.369	<.01	0.770
GTL	MLL	0.094	9.306	<.01	0.188
GA	MLL	0.010	19.997	<.01	0.819

According to the results of the structural regression model presented in Table 5, the effects of the independent variables on the dependent variables were found to be positive and statistically significant. These effects show varying strengths across different variables. The effect of GA on F (β = 0.600) indicates a strong relationship, showing that GA has a significant impact on F (p < .01).

In contrast, the effect of GTL on F is moderate (β = 0.285), indicating a relatively lower influence compared to GA.

The effect of GA on E (β = 0.501) is moderate to high, indicating a strong relationship, whereas the effect of GTL on E is weaker (β = 0.113). The effect of GA on IE (β = 0.770) is quite strong, representing one of the highest impacts in the model. This demonstrates that GA is a decisive factor in determining IE scores.

The effect of GTL on MLL (β = 0.188) is low to moderate. However, the effect of GA on MLL (β = 0.819) is extremely high, indicating that GA has a much stronger influence on MLL compared to the other independent variable.

Overall, when examining the standardized coefficients, GA shows the strongest effects on IE and MLL, while GTL appears to have a more moderate effect, particularly on F (β = 0.285). GA generally exerts higher levels of influence on the dependent variables, whereas GTL is more effective specifically on F and MLL. These results suggest that the independent variables play distinct and significant roles on the dependent variables in different contexts.

After constructing the model and examining the path coefficients and variable relationships, fit indices were calculated to determine how well the model fits the data. The fit index values are presented in Table 6.

Table 6
Model Fit Indices

Fit Index	*Value*	*Fit*
CFI	1.000	Excellent Fit
TLI	1.000	Excellent Fit
RMSEA	0.000	Excellent Fit
SRMR	0.000	Excellent Fit

The CFI (Comparative Fit Index) value presented in Table 6 being close to 1 indicates that the model has an excellent fit. Similarly, the TLI (Tucker-Lewis Index) value also suggests excellent fit and that the model fits the data very well. The RMSEA (Root Mean Square Error of Approximation) value of 0.000 reflects a very low approximation error, which is an indicator of good fit. The SRMR (Standardized Root Mean Square Residual) value of .000 also indicates that the model fits the data very well. Based on these fit indices, it can be concluded that the model is well-validated, and therefore the relationships and effects among the variables are statistically significant.

4. Discussion, Conclusion, and Recommendations

This study investigated in detail the relationship between mathematical literacy and both geometry achievement and geometric thinking levels, while also examining the influence of these two variables on mathematical literacy and its sub-processes. Through correlation analysis and structural equation modeling, the relationships among students' geometry achievement, geometric thinking levels, and the space and shape domain of mathematical literacy and its sub-processes were revealed. In this context, significant positive relationships were found between GA, GTL, and the sub-processes of F, E, and IE, as well as with the space and shape domain of MLL (Mathematical Literacy Level). The findings highlight the importance of incorporating problem types involving formulating and interpreting/evaluating skills more frequently into geometry instruction to enhance students' success in mathematical literacy.

The relationship between GA and the space and shape domain of MLL was found to be stronger than the relationship between GTL and the same domain. Among the sub-processes of mathematical literacy, GA showed the highest correlation with F. Similarly, GTL had the strongest relationship with the F sub-process. The strong positive correlation between GA and MLL (particularly in the space and shape domain) aligns with findings in the literature, such as those by Battista (2007) and Uttal et al. (2013), who emphasized the contribution of geometric skills to mathematical achievement. In particular, Uttal et al. (2013) emphasized that geometric abilities are critical for conceptual development in mathematics, which is consistent with the high correlation observed in this study.

The structural equation modeling used in this study clarified the extent to which the examined variables influenced one another. In this regard, GTL was found to have a moderate effect on MLL. However, GA demonstrated a very strong impact on MLL, indicating that GA plays a significantly more influential role compared to the other independent variable. This finding may be interpreted as the result of GA involving not only conceptual knowledge but also procedural skills, which are directly related to student success in context-based mathematical problems. In contrast, GTL may not directly reflect on contextual interpretation and problem-solving components of mathematical literacy. Therefore, GA appears to play a more dominant role in explaining the multidimensional structure of MLL.

GA also had a very strong impact on the IE process, representing one of the most significant effects in the model. GTL, on the other hand, was found to have a moderate effect primarily on the F process. This suggests that students' geometry knowledge supports not only their ability to calculate but also their ability to explain mathematical results and evaluate them in context. Meanwhile, GTL had its strongest impact on the F process. This can be explained by the fact that as students progress through the Van Hiele levels, they become more proficient and deliberate in recognizing and distinguishing shapes and relationships (analysis), making connections among properties, and drawing generalizations (informal deduction). These types of skills form the foundation of transforming a problem into a mathematical model. Koedinger and Nathan (2004) identified that students face cognitive difficulties in translating verbal problems into mathematical expressions, and that this transformation process is a key determinant of mathematical success. This transformation corresponds to the "formulating" skill as defined in the PISA framework. Therefore, the high correlation between the F sub-process and both GA and GTL in this study suggests that this skill is directly related to students' ability to convert problem contexts into mathematical structures.

Geometry instruction has been shown to significantly enhance students' analytical thinking, abstraction, and logical reasoning skills (Sinclair & Bruce, 2015). Clements and Sarama (2011) also emphasized the critical role of geometric knowledge in problem-solving and reasoning processes. Skills such as problem-solving, reasoning, and deduction are essential components of mathematical literacy. Hence, the association between geometry achievement and these skills directly affects students' mathematical literacy success. This study's results are consistent with findings in the existing literature.

Geometric thinking skills directly influence problem-solving, spatial reasoning, and mathematical reasoning processes (Van Hiele, 1986). Studies in the literature indicate that students with high levels of geometric thinking perform better in conceptual understanding, mathematical reasoning, and problem-solving (Van Hiele, 1986; Sinclair & Bruce, 2015; Clements & Sarama, 2011). Similarly, this study found that as students' geometric thinking levels increase, their understanding of abstract mathematical concepts and ability to construct mathematical relationships improve, which in turn enhances their MLL.

This study also presents some practical implications. Instead of relying solely on information transfer in geometry education, instructional activities should support students' visual-spatial thinking, analysis, generalization, and logical inference skills. In particular, adopting a gradual teaching process based on the Van Hiele model and providing content tailored to students' geometric thinking levels can enhance both geometry achievement and literacy levels.

The strong influence of GA on MLL found in this study suggests that geometry should be regarded not just as a means for conveying conceptual knowledge, but also as a strategic tool for developing literacy skills. Accordingly, geometry classes should incorporate more activities that promote high-level thinking skills such as problem-solving, modeling, argumentation, and interpretation. Moreover, assessment tools designed to improve mathematical literacy should be structured not only to evaluate students' ability to reach correct answers, but also to assess their strategies, interpretive capacity, and use of mathematical language throughout the process. This approach will help students become better prepared for both national and international assessment systems.

However, this study has several limitations. Primarily, the sample is limited to a specific region and a relatively small group of middle school students, which should be taken into account when generalizing the findings. Future research should explore these relationships in more depth across different age groups and educational levels to support the development of more effective educational interventions. Additionally, long-term longitudinal studies would provide insight into how these relationships evolve over time, helping to ground educational strategies on firmer foundations. Such studies would also support the development of critical policies aimed at improving the quality of mathematics education.

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