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Research Article

The effect of inquiry based technology integration on conceptual and procedural geometry knowledge of preservice mathematics teachers

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This study was intended to investigate the effect of a technology-integrated guided inquiry-based learning and guided inquiry approach on preservice mathematics teachers' plane geometry conceptual and procedural knowledge. For this purpose, a pretest-posttest quasi-experimental design group was employed. A total of 116 PSMTs participated in three intact groups: EG1 (n = 48) treated with TGIBL, EG2 (n = 38) with GIBL and CG (n = 30) in traditional approach. The data were collected using a two-tiered geometry test and a geometry procedural knowledge test, with reliability KR20 =.87 and ICC =.879, respectively. A one-way MANOVA and paired t-test were used to analyze the data. Findings indicated a statistically significant difference between the groups, and PSMTs taught with technology-integrated guided inquiry significantly achieved more geometry conceptual and procedural knowledge than other groups. Therefore, it is important that teacher educators consider a technology-integrated guided inquiry approach to improve PSMTs' conceptual and procedural knowledge of geometry at the college of teacher education.

Keywords: Conceptual knowledge, procedural knowledge, guided inquiry, technology, geometry

1. Introduction

In this century, having a strong foundation of mathematics provides learners with the means to understand, communicate, and transform the world. It serves as a basis for science and technology and as a tool that determines opportunities and possibilities for people's futures (National Council of Teachers of Mathematics [NCTM], 2000; Ministry of National Education [MoE], 2020). Thus, it is crucial to study mathematics in a way that makes senses (Hiebert & Carpenter, 1992; Kilpatrick et al., 2001). In the mathematics curriculum, geometry is an important part of the curriculum and is used in science and technology, mathematical modeling, problem-solving, and representing real-life problems (Clement & Sarama, 2011; Luneta, 2015; Marchis, 2012). For instance, geometrical ideas like triangles, circles, rectangles, lines, squares, areas, and perimeters are employed in practical application such as building, surveying, traffic sign, engineering and other applications (Siyepu & Mtonjeni, 2014).

To this effect, it is essential to learn geometry with understanding (Hiebert & Carpenter, 1992). As a result, in order to address the demands of their pupils, PSMTs must possess mathematical competences. However, understanding geometric concepts requires not only mastering certain basic concepts or abilities but also creating connections between concepts. According to Siregar and Siagian (2019), geometry makes sense for PSMTs if they understand the relationships between the geometric concepts. Based on Skemp's theory (1978), there are relational and instrumental understandings. However, Hiebert and Carpenter (1992) referred to these types of knowledge as conceptual and procedural knowledge. Conceptual knowledge describes a network and interconnection of concepts whereas procedural knowledge describes a step-by-step procedure in solving mathematical problems.

Conceptual knowledge is web of concepts which are the foundation for mathematical structure, concept connections, and the interpretation of mathematical procedures. According to Rittle-Johnson et al. (2015), conceptual knowledge is knowledge of abstract ideas, including understanding of concepts and their interrelation. Similarly, Star and Stylianides (2013) stated conceptual knowledge as a mathematical knowledge involving principles and definitions. PSMTs obtained conceptual knowledge if they can state and define logical relationships between concepts (Mariquit & Luna, 2017). Thus, conceptual knowledge is not limited to an ability to memorize the meaning of concepts, but can explain concepts, relationships between concepts, how and why concepts related.

As a result, conceptual knowledge in geometry include understanding the meaning of geometric objects (e.g., parallel lines, angles, plane shapes, etc.), classifying based on their geometric properties, understanding the relationship between concepts (e.g., equilateral with an isosceles triangle, rectangular relationship with a parallelogram, similarity, congruence, etc.), and knowing how to figure out formulas (e.g., perimeter, area, etc.). In this condition, if mathematics teachers emphasis on the links between geometrical concepts, then PSMTs attain an interconnected comprehension of geometrical concepts (Evitts, 2005).

Procedural knowledge is also another important piece of mathematical knowledge. According to Star and Stylianides (2013), it is knowledge of rules, routines, a fixed set of procedures, and symbolic notations implied in problem-solving. This indicates that procedural knowledge is a mastery of computational abilities and knowledge of procedures for application in mathematical components, algorithms, and definitions. Furthermore, Rittle-Johnson et al. (2015) and Star (2005) argue procedural knowledge is not only limited to the knowledge of procedures but also knows how to obtain them.

Studies have described conceptual knowledge and procedural knowledge in different ways, like their relationship, interaction, development, and significance (Hapaasalo & Kadijevich, 2000; Rittle-Johnson et al., 2001; Star, 2005). According to Hiebert and Lefevre (1986), both mathematical and geometric knowledge are two key components that are positively associated and cannot be absolutely disconnected. In addition, Areaya and Sidelil (2012) stated that both conceptual and procedural knowledge are distinct but related. Thus, this indicates that flexible instruction in which conceptual knowledge serves as a source for procedural knowledge and vice versa is significant during mathematics classroom instruction.

Studies at the national and international level have shown that PSMTs' geometry learning outcomes have been low (Fujita & Jones, 2006; Kasa, 2015; Marchis, 2012; MoE, 2017). For instance, the study by Marchis (2012) in the UK showed pre-service primary school teachers' geometric knowledge is inadequate, and they couldn't recognize basic geometrical shapes. Similarly, Kasa (2015) and MoE (2020) showed that the geometric knowledge of PSMTs in Ethiopian colleges of education is below standards. Furthermore, Fyfe et al. (2015) stated that geometric difficulties are the main underlying problems in learning other mathematical concepts.

Accordingly, traditional teacher-centered approach may not be effective method to prepare PSMTs for meaningful learning (Sebsibe & Feza, 2019; Yimer, 2020). Thus, new and innovative approach that supports PSMTs with meaningful learning of geometric concepts should be investigated. One alternative to the traditional way of teaching is Guided inquiry-based learning (GIBL) (Laursen & Rasmussen, 2019).

GIBL approach is a learner-centered approach in which PSMTs actively engage, and process information at a higher level with the provision of the classroom teacher (Darkis, 2020; Lazonder & Harmsen, 2016; Sebsibe & Feza, 2019). In this case, PSMTs construct their own understanding based on prior knowledge within collaborative activities. However, GIBL is more extensively applied in the science classroom than in the mathematics classroom (Caswell & Labrie, 2017; Gardner, 2012).

Besides, because geometry involves abstract concepts, educational technologies need to be integrated into the teaching and learning processes for visualizing these abstract concepts (Wong et al., 2011). Alternatively, literatures also suggest that technology-supported learning approaches can improve the teaching and learning process if it is integrated within a meaningful method (Atnafu et al., 2015; Charles-Ogan & George 2015; Eshetu et al, 2022; Gemechu et al., 2018; Sebsibe & Feza, 2019). In a technology-integrated guided inquiry-based learning [TGIBL] approach, PSMTs engage in higher thinking levels (Eshetu et al., 2022; Fantu, 2014).

Although the TGIBL teaching approach has received considerable attention from educators and policymakers, there aren't many programs for training future teachers to employ technology-advanced materials to improve inquiry-based learning (Gerard et al., 2011). In the same manner, studies have revealed that an instructional shift is yet to be practiced in CTEs in Ethiopia in teaching mathematics and geometry, which remains an open question (MoE, 2012a; 2015; 2020). Thus, the current study is intended to contribute to this limited research base by reforming the teaching of plane geometry at CTEs. Therefore, this study was initiated to examine the impact of TGIBL and GIBL methods on PSMTs' conceptual and procedural knowledge of plane geometry. Thus, the study has important implications for preservice mathematics teacher education by highlighting optimal learning settings and approaches. For this purpose, the 5E (**E**ngage-**E**xplore-**E**xplain-**E**laborate-**E**valuate) lesson plan was used. Hence, the current study tests the following hypotheses:

H1) There is no significant mean difference of geometric conceptual and procedural knowledge among the groups.

H2) There is no significant mean difference between the pretest and post-test of geometric conceptual and procedural knowledge in each group.

2. Literature Review

Despite various perspectives, constructivism emphasizes knowledge being constructed in a learner-centered instructional approach rather than passively absorbed. This deviates from the teacher-centered approach in which the knowledge is imparted by the teacher. In a constructivist classroom, the teacher serves as a facilitator while learners actively construct knowledge by engaging and interpreting ideas from social and personal experiences as well as prior knowledge (Abiatal & Howard, 2020).

The GIBL approach is an application of constructivist theory, advocating that learning should be active and knowledge constructed. Kandil and Işksal-Bostan (2019) added that GIBL needs PSMTs to ask questions, look for solutions, and build a connection to prior knowledge rather than just gather data. Instead of memorization, PSMTs inquire using materials and solve problems by questioning, exploring, observing, discovering, and proving (Artigue & Baptist, 2012; Dorgu, 2016). According to Saunders-Stewart et al. (2012), GIBL improved PSMTs' achievement, application of knowledge, critical thinking, and problem-solving abilities, as well as attitudes.

Furthermore, Mostert and Clark-Wilson (2016) stated that technology makes it simpler for teachers to deliver instructional resources with precise mathematical content and visuals. Technology gives PSMTs opportunities to explore mathematical concepts in innovative ways. The TGIBL approach allows PSMTs and instructors to visualize abstract geometric concepts for achieving higher-level thinking, problem-solving skills, and reasoning about plane geometry (Bokosmaty et al., 2017). Hence, dynamic geometry software such as Geometric Sketchpad [GSP], GeoGebra, etc. is some of the technology that supports representing geometrical concepts.

The GSP is simple to use and promotes the inquiry method, in which PSMTs can visualize and understand a geometric problem before formulating hypotheses and trying proofs (Hulme, 2012). In order to formulate and conjecture, and build geometric objects, GSP has been widely utilized by instructors, mathematics educators, and students at both the high school and college levels (Guven & Kosa, 2008; Meng & Sam, 2013; Oldknow et al., 2010).

Furthermore, GSP has a positive impact on learner learning outcomes in geometry, algebra, pre-calculus and calculus (Adelabu et al., 2019; Arbain & Shukor, 2014; Kotu & Weldeyesus, 2022; Zulnadi & Zamri, 2017). In her class of discrete mathematics, Quinn (1997) discovered that GSP is a crucial teaching tool for graph theory and PSMTs were able to verify or disprove certain graphs-theory concepts.

3. Method

3.1. Research Design

This study employed a non-equivalent quasi-experimental research design. This design was employed since there were both treatment and comparison groups in the study (Creswell, 2012). In this form of research design, the non-random distribution of subjects is used as a treatment and control group. In this case, a quasi-experimental design was employed to determine the causal effect of an intervention on its target population, in which the outside factors were tightly controlled (White & Sabarwal, 2014). Therefore, the intervention was conducted in their intact classes, where PSMTs were assigned to experimental and comparison groups. The experimental group consisted of two classes, namely the first experimental class [EG1] and the second experimental class [EG2]. The EG1 group was treated with a TGIBL approach, while the EG2 group was treated with a GIBL approach. On the other hand, the conventional approach was employed in the comparison group. Table 1 shows the design outline.

Table 1
Outline of research design

Group		*Interventions*	
Experimental Group 1 (EG1)	Pretest	TGIBL	Posttest
Experimental Group 2 (EG2)	Pretest	GIBL	Posttest
Comparison Group	Pretest	-	Posttest

3.2. Setting, Population and Sampling

This research was conducted in Ethiopia, Oromiya Regional State, at CTEs. The population of the study was all PSMTs of the 2020 academic year who were registered for Math-111 (Plane Geometry). A total of 116 PSMTs in which (#EG1 = 48), (#EG2 = 38) and (#CG = 30) were participated in the study. In the study, the three-stage sampling technique was employed. To begin with, two CTEs (Dambi Dollo and Shambu) were selected purposefully based on their equivalency in ICT facilities, academic staff, candidate enrolment, and demography. Purposive sampling is used when the researcher believes that useful data can be obtained under specific conditions (Fraenkel & Wallen, 2009). Following that, simple random sampling was employed to assign the two CTEs into experimental and comparison groups. Thus, Dambi Dollo CTE was allocated for treatment while Shambu CTE was sampled for comparison. Finally, the intact classes from Dambi Dollo CTE were assigned to EG1 (technology-integrated guided inquiry) and EG2 (guided inquiry) using a simple random sampling method. The experimental groups were in opposite shifts, so when one group left class for practicum, the other group stayed in CTE for intervention.

3.3. Instruments

The CK and PK of PSMTs were assessed using a two-tiered geometry diagnostic test [TTGDT] and a geometry procedural knowledge test [GPKT], respectively. It is a non-routine question designed to measure PSMTs' conceptual and procedural knowledge. The scoring method for the TTGDT was 1 mark if both tiers were correct, otherwise 0; for PK, the rubric was adapted, and the analysis procedure was determined based on the scoring rubric and the score of each response to the question. Table 2 shows the methods of the scoring rubrics.

Each item was scored using the criteria indicated in Table 2, and the total scores that the PSMTs received from the test were calculated using these scores. The TTGDT have 30 items and GPKT have 10 items. The highest and lowest scores for the CK score are 30 and 0, respectively while for PK score are 30 and 0 respectively. The pilot study was conducted at CTE with forty PSMTs, who were not selected for the main study and this satisfies the minimum requirements for pilot study (Bujang & Baharum, 2017).

Table 2
The scoring method of TTGDT and GPKT

Geometry conceptual knowledge test [TTGDT]			*Geometry procedural knowledge test [GPKT]*	
First tier	Second tier	Marks	Criteria and Descriptions	Marks
Correct	Correct	1	A correct and complete answer.	3
Incorrect	Correct	0	The response is partially correct and addresses most aspects of the task, using mathematically sound procedures.	2
Correct	Incorrect	0	This response is incomplete and exhibits many flaws but is not completely incorrect; it addresses some elements of the task correctly but reaches an inadequate solution and provides reasoning that is incomplete.	1
Incorrect	Incorrect	0	When the response is incorrect or not attempted	0

Note. Adapted from Francis (2019), and Ndlovu and Mji (2012).

3.4. Validity and Reliability

The validity of the instrument in the study relied on expert judgments. Thus, the content and face validity of the instrument were checked by professionals from mathematics education, school of measurement, and evaluation (Cohen et al., 2007; Kothari, 2004). Then, some items were revised depending on the feedback and comments from these professionals on the clarity and errors in the answer keys. Similarly, the reliability of the instrument in the study used to measure the degree of internal consistency to complement the validity (Fraenkel &Wallen, 2009).

The inter-rater agreement or inter-rater reliability was used to measure the reliability of GPKT items. In this study, thus, one mathematics teacher educator and the researcher were to rate the items based on the criteria of the scoring rubrics in Table 3.2. The inter-reliability for the GPPT items was measured at a correlation coefficient of .87 (Liao et al., 2010), and the inter-rater agreement reliability (intra-class correlation/ICC average) of .87 shows a high value (Graham et al., 2012). If the ICC is greater than .75, the reliability is considered acceptable (Leech et al., 2012).

On the same manner, the KR20 was used to measure the reliability of TTGDT items. Based on Kuder-Richardson's 20 [KR20], the reliability was determined to be .87. This indicates that the value is within acceptable ranges. In addition, the psychometric measure of mean difficulty index is .49 and discrimination index is 0.33 shows within acceptable ranges (Boopathiraj & Challamani, 2013). Thus, the test items for both CK and PK are good and were used for the actual study.

3.5. Procedure

Before the interventions were conducted, the researcher discussed with mathematics teacher educators and identified that they have computer knowledge but are unfamiliar with the usage of Dynamic Geometry Software GSP for an instructional purpose. The mathematics teacher educators with master’s degrees and nine years of teaching experiences in CTEs were recruited for the interventions and comparison groups.

Following that, training was provided to mathematics teacher educators assigned for the experimental groups on how to implement interventions, apply computer-assisted learning packages (GSP, GeoGebra) in guided inquiry-based learning settings, and guide the use of inquiry learning strategies. The training lasted for five days and included an overview of the TGIBL and GIBL approaches and how to implement 5E, which have five phases (Engage-Explore-Explain-Elaborate-Evaluate) lesson plans. The interventions were implemented for a whole semester (12 weeks) for four hours per week. During the beginning of the lesson, PSMTs in the experimental groups were divided into groups of four to five, based on the classroom teachers' comments and their academic abilities (see Appendix 1).

The Following shows specific practices in each Experimental and comparison groups:

Experimental Group I (EG1)/Technology-Integrated Guided Inquiry. PSMTs were divided into four-to-five members of heterogeneous groups. In this approach, activities that involve PSMTs to explore, investigate, discover, reflect on, and visualize the geometrical concepts, manipulating, animating, and dragging them, and analyzing and making conjectures using technology (GSP or GeoGebra), were given. In addition, YouTube videos were used. The instruction was based on the 5E lesson plan.

Experimental Group II (EG2)/Guided Inquiry-Based Learning. PSMTs were grouped into members of heterogeneous groups. In this approach, hands-on activities and manipulatives are used. In this case, concrete materials such as manipulatives were used to explore and investigate, conjecture geometric properties and concepts, and connect and model these properties and concepts through cutting, folding, pasting, connecting, and modeling activities. Hence, PSMTs are working collaboratively to think-pair, discuss, and investigate the activities given. In the meantime, teacher educators challenged PSMTs by asking questions like, "How can you be sure it is correct? What is your explanation for what you have got? This elicits PSMTs to make further explanations and evaluations based on the collected evidence.

Comparison Group/Conventional Teacher-Centered Method. In a comparison group, the conventional lecture method was used. The mathematics teacher educator followed the usual trend of the traditional lecture approach used in higher educational institutions in Ethiopia for their instructional tasks. In this trend, the mathematics teacher educator presented the content and worked on some sample examples while PSMTs took notes, passively listened to the lectures, and copied formulas. In this approach, no manipulatives, real-life examples, or technologies were used.

In addition, during the process of the interventions, the researchers followed its implementation process and provided feedback and comments at the end of classroom activities for further improvement of the interventions. Finally, after the completion of the interventions, a post-test was given to all groups.

3.6. Data Analysis

The analysis was performed with one-Way Multivariate Analysis [MANOVA] because it has three factors namely TGIBL, GIBL and conventional with two continues dependent variable and three different groups of participants involved could be taken as the other factor for using these inferential statistical tools at ($\alpha = .05$) significant level (Cohen et al., 2007; Field, 2009). In addition, a paired sample *t*-test is employed to see the changes of mean gain between the pretest and posttest for each group.

Every statistical test, whether it is parametric or non-parametric, starts with a number of assumptions about the data that it will be applied. However, parametric tests are regarded powerful than non-parametric test in sensing the differences existing between the groups and this is tenable only if the assumptions like normality of the distribution, homogeneity of variance, and independence of samples are met (Field, 2009). Thus, these assumptions must be checked for their validity before to conduct the analysis. The observation of sample independence was not violated because each PSMT in study did their own test by themselves independently.

Of the aforementioned assumptions, normality should be verified before conducting any inferential statistics. If any of the independent samples' normality is violated, parametric testing shouldn't be used (Rietveld & van Hout, 2015). The skewness and kurtosis measurements were used to check whether the data for each of the groups were normally distributed (Field, 2009). According to Field (2009), the Kolmogorov-Smirnov test is not useful and is less accurate in practice. The normality of the given data is attained if the skewness and kurtosis values are between –2 and 2 (George & Mallery, 2003).

4. Results

Table 3 shows the skewness and kurtosis of the pretest and posttest scores of PSMT's CK and PK scores.

Table 3
The normality distribution of CK and PK scores

			Statistics							
			Pre-test				Post-test			
DV	*Group*	*N*	*M*	*SD*	*Skewness*	*Kurtosis*	*M*	*SD*	*Skewness*	*Kurtosis*
CK	TGIBL	48	17.58	5.58	−.364	−.394	27.38	6.70	−.540	−.472
	GIBL	38	18.00	4.18	.142	−.813	23.79	4.63	.050	−1.326
	CG	30	17.77	4.89	−1.066	1.876	21.13	6.89	−.464	−.167
PK	TGIBL	48	15.33	5.62	.738	−.799	26.91	2.74	−.707	−.448
	GIBL	38	15.21	4.48	.596	−.810	23.34	2.69	−.246	.094
	CG	30	16.03	5.50	.346	−1.481	21.02	6.89	.543	−.042

Note. CK = Conceptual knowledge; PK = Procedural knowledge.

Each skewness and kurtosis value shown in Table 3.3 for the distribution of EG1, EG2, and CG on pretest and posttest scores for CK and PK was in support of normality as those values are between −2 and +2 (George & Mallery, 2003). Therefore, parametric test is used for further analysis of data.

The intent of the pretest is to determine whether there is a difference between the three groups on geometry CK and PK before the commencement of the interventions. The primary hypothesis of the study was that the intervention groups would be comparable. Thus, according to Wiersma and Juurs (2005), the researcher must look at the homogeneity of the intervention groups before applying the instruction. Table 4 shows the ANOVA test pretest scores of the groups.

Table 4
A one-way ANOVA test for CK and PK pretest scores

DV	*Sources*	*SS*	*Df*	*MS*	*f*	*Sig*
CK	Between Groups	3.682	2	1.841	.074	.93
	Within Groups	2807.033	113	24.841		
	Total	2810.716	115			
PK	Between Groups	12.973	2	6.487	.236	.79
	Within Groups	3105.949	113	27.486		
	Total	3118.922	115			

From the Table 4, it is understood that the ANOVA results revealed that there was no statistically significant mean difference between the groups in terms of CK and PK scores before the interventions (Pallant, 2011). Therefore, it can be deduced that the difference observed on the posttest score is attributable to the implementation of the interventions.

HO $_{[1]}$: *There is no significant mean difference between the mean scores of geometry CK and PK among the three groups.*

To answer the hypothesis MANOVA test was conducted. In this study, since there were two dependent variables and three groups, MANOVA is the best way to test the differences among groups in terms of dependent variables than ANOVA (Pallant, 2011). The MANOVA can deal with several dependent variables, whereas the ANOVA can only deal with one dependent variable. Prior to conduct MANOVA, necessary assumptions such as independence of observation, normality, outliers, multivariate homogeneity, multicollinearity and singularity must be checked. As shown in Table 3 the normality of CK and PK posttest scores in each group not violated.

The outlier was checked using Mahalanobis distance for posttest data (Pallant, 2011). For this study, two posttest dependent variables (CK and PK) were used. The reported the critical value for two dependent variables is 13.82 (Pallant, 2011). Table 5 shows Mahalanobis distance tests.

Table 5

Mahalanobis distances for posttest scores

Variables		*N*	*Min*	*Max*	*Mean*	*SD*
Posttests	Mahal. Distance	116	.039	8.079	1.983	1.712

As seen from Table 5, the Mahalanobis distance for post test data of this study was 8.079. This is below the critical value for two dependent variables which is 13.82. So, there was no outlier for post test data. Hence the assumption of outlier was not violated.

In order to check for multivariate homogeneity of variance-covariance, Box's Test of equality of co-variance was employed. The following Table 6 shows Box's Test of equality of covariance.

Table 6

Box's Test of Equality of Co-variance Matrices for posttest score

	Box's Test of Equality of Covariance of Matrices [a]				
Variable	*Box's M*	*f*	*df1*	*df2*	*Sig.*
Posttest	10.42	1.69	6	154515.296	.119

Note. a. Design: Intercept + group

From Table 6, it can be deduced that homogeneity of variance-covariance matrices was not violated as the Box's M value is not significant for posttest (p >.001) (Field, 2009; Pallant, 2011). Hence, Wilks' Lambda is an appropriate test to use than other such as Pillai's Trace, Hoelling's Trace, etc.

In addition, the assumption of multicollinearity and singularity was checked. This assumption was checked using Pearson Correlation. The assumption is violated if the correlation between the variables is r >.8 (Field, 2009; Pallant, 2011). The correlation between CK and PK is r =.333. Hence, the multicollinearity assumption is not violated. Furthermore, Levene's test was conducted to examine the homogeneity of variances between the groups. Table 7 shows the Levene's tests.

Table 7

Levene's Test of Equality of Variances of posttest score

	Levene's test of Equality of Error Variances [a]			
Variables	*f*	*df1*	*df2*	*Sig.*
CK	1.701	2	113	.187
PK	.219	2	113	.803

Note. a. Design: Intercept + group

From the Table 7 above, it can be seen that (p > .05) for both variables. Hence, the homogeneity of variance is not violated (Field, 2009). This indicated that the homogeneity of variance between is similar. After all the assumptions were checked, MANOVA test was conducted to assess if there were statistically significant posttests mean score differences between the three groups. The following Table 8 shows the MANOVA test.

Table 8

The MANOVA test of posttest score

	Wilks' λ	*f*	*df*	*Error df*	*Sign.*	η^2
Groups	.527	21.167[b]	4	112	.000	.274

Note. b. Exact statistic

As revealed from the Table 8, the MANOVA test exposed that there was a statistically significant difference between the three groups on posttest mean scores, *Wilks'* λ= .527, F (4, 112) = 21.167, p <.001; multivariate η^2 = .274. The eta squared (η^2) showed large effect and 27.4 % of multivariate variance of posttest mean scores was associated with the intervention (Cohen, 1988). This means that the difference between the groups accounted for by the intervention.

Following the MANOVA test, we examined the univariate ANOVA results. The ANOVA (Between Subjects) was computed to determine whether the groups differ on each of these variables, examined alone. The ANOVs also help us understand which variables, separately, differ across groups (Field, 2009). The following Table 9 shows the univariate ANOVA tests.

Table 9

Tests of between-subject effects

IV	*DV*	*Type III SS*	*df*	*MS*	*f*	*Sig.*	η^2
Groups	CK	755.105	2	377.553	9.980	.000	.150
	PK	683.490	2	341.745	44.819	.000	.442
Error	CK	4275.032	113	37.832			
	PK	861.622	113	7.625			

As shown in the Table 9, univariate ANOVAs indicated that both geometry conceptual knowledge and procedural knowledge were significantly different for PSMTs with different intervention, F (2, 113) = 9.98, p < .001, eta squared (η^2) = .15 and F (2, 113) = 44.819, p <.001, eta squared (η^2) = .442, respectively. The eta squared (η^2) values are .15, .442 shows large effect (Cohen, 1988) indicating that 15% and 44.2% variation of CK and PK respectively was associated with treatment. Furthermore, post-hoc multiple comparisons were performed to determine which group differed from the others. In this case, a Bonferroni type adjustment should be made in order to ensure a lower type-I error on multiple comparisons (Tabachnick & Fidell, 2007). Table 10 shows post-hoc multiple comparisons.

Table 10

Post-hoc multiple comparison tests

DV	Group (I)	Group (J)	Mean Difference (I-J)	SE	Sign. [b]	95% CI for difference [b] LB	UB
CK	TGIBL	GIBL	3.586*	1.336	.020	.34	6.831
		CG	6.242*	1.432	.000	2.763	9.720
	GIBL	CG	2.656	1.502	.239	−.994	6.307
PK	TGIBL	GIBL	3.564*	.600	.000	2.107	5.021
		CG	5.890*	.643	.000	4.328	7.451
	GIBL	CG	2.325*	.674	.002	.687	3.964

Note. *. The mean difference is significant at the .05 level; b. adjustment for multiple comparisons: Bonferroni.

As shown in the Table 10, the post-hoc multiple comparison result revealed that there was statistically significant mean difference between each pair of PSMTs group in TGIBL and GIBL (p =.02) and TGIBL and CG (p <.001) but not between GIBL and CG (p =.239) in geometry conceptual knowledge mean scores. Similarly, there was statistically significant mean difference between each pair of PSMTs groups in TGIBL and GIBL (p <.001), TGIBL and CG (p <.001) and GIBL and CG (p =.002) in geometry procedural knowledge mean scores. Therefore, the PSMTs in technology integrated guided inquiry approach outperformed other groups in geometry conceptual and procedural knowledge. This indicates that technology supported approach has more effects than guided inquiry and traditional approaches.

HO $_{[2]}$: *There is no significant mean difference between the pre-test and post-test mean scores of geometry CK and PK in each group.*

Likewise, based on the mean gains between the pre-test and post-test scores for each group, the geometry CK and PK of PSMTs in three groups were compared using a paired sample t-test. The results are displayed in Table 11.

Table 11
A paired sample t-test of pre-post score of geometry CK and PK for the three groups

			Paired difference		95% CI				
DV	Group	Observations	*MD*	*SD*	*LB*	*UB*	*t*	*df*	*Sig.*
CK	TGIBL	Post – Pretest	9.792	6.591	7.8776	11.7056	10.29	47	.000
	GIBL	Post – Pretest	5.789	4.503	4.3093	7.2696	7.93	37	.000
	CG	Post – Pretest	1.333	4.257	−.45615	2.7228	1.46	29	.156
PK	TGIBL	Post – Pretest	11.573	6.338	9.7326	13.4133	12.65	47	.000
	GIBL	Post – Pretest	8.132	4.655	6.6014	9.6617	10.77	37	.000
	CG	Post – Pretest	2.733	4.514	1.0478	4.4188	3.32	29	.002

As shown in Table 11, a paired-samples t-test revealed that PSMTs' conceptual test score improved significantly from pre-intervention (pre-test) to post-intervention (post-test) (t (47) = 10.29, p <.001, d = 1.485) for the TGIBL group, and the effect size was very large (Cohen, 1988), but not for the comparison group (t (29) = 1.46, p =.156). The PSMTs in GIBL also made a significant improvement on the conceptual test score (t (37) = 7.93, p <.001, d = 1.285), and the effect size is very large (Cohen, 1988). In terms of procedural knowledge, the paired sample t-test result confirmed that the procedural test score of PSMTs in all groups were made significant improvements from pretest to posttest.

5. Discussion

The main objective of the study was to examine the effect of a technology-integrated guided inquiry approach on the conceptual knowledge and procedural knowledge of geometry of PSMTs who studied a plane geometry course (Math-111) in CTEs. A one-way MANOVA test was employed to analyze the difference between the three groups of PSMTs on the dependent variables CK and PK. The finding showed that there was a significant mean difference between the groups of PSMTs on geometry conceptual and procedural knowledge.

The follow-up univariate ANOVAs indicated that both conceptual and procedural knowledge were significantly different for three groups of PSMTs, F (2, 113) = 9.98, p < .001, η^2 = .15 and F (2, 113) = 44.819, p < .001, η^2 = .442, respectively. The eta squared values are indicating large effect size (Cohen, 1988) that 15% and 44.2% variation of CK and PK respectively was associated with the interventions. To this effect, the study revealed that PSMTs who taught geometry using the TGIBL approach outperformed both PSMTs who were taught using the GIBL and conventional methods of instruction on attaining geometry conceptual and procedural knowledge. This study supported previous research findings by Yusuf and Afolabi (2010), Zulnaidi and Zakaria (2012), Donevska-Todorova (2015), Murni and Jehadus (2019), Salifu et al. (2020), Tezer and Cumhur (2017), Cesaria and Herman (2019), Yimer (2020), and Kotu and Weldeyesus (2022), in which learners taught using computer-based instruction outperformed those taught using non-computer-based instruction.

GSP integrated with guided inquiry approach can be used in teaching and learning geometry to enhance conceptual and procedural knowledge among PSMTs in CTEs. According to Ocal (2017), and Zulnaidi and Zamri (2017) geometric software are useful for developing conceptual understanding, critical thinking, reasoning skills and procedural knowledge. In addition, instructional technologies are important for multiple representation and visualizations of abstract geometric concepts which making them easier to learn than traditional teacher-centered approach (Donevska-Todorova, 2015). Furthermore, Salifu (2020) claimed that PSMTs who learned the circles theorem utilizing technology achieved a considerably higher mean score than those who were taught using the traditional method.

On the other hand, the findings of the study also showed PSMTs who were taught with guided inquiry performed better in geometry procedural knowledge than those in the traditional approach, while there was no significant difference in geometry conceptual knowledge between groups. This finding is similar with the studies by Dagnew and Mekonnen (2020) and Kandil (2016), who were found that guided inquiry approaches have a positive impact on mathematics learning outcomes. Guided inquiry-based learning empowers PSMTs to own their education and promotes independent thought (Magee & Flessner, 2012; Tofel-Grehl & Callahan, 2014). Through these interactions, PSMTs gain both procedural and deep conceptual understanding (Lewis & Estis, 2020). In addition, PSMTs benefit from guided inquiry-based learning by developing processing skills, a thorough knowledge of mathematical concepts, and retentions of geometric concepts (Celikten et al., 2012).

Similarly, the paired sample t-test results revealed a significant mean gain in geometry conceptual and procedural knowledge for both technology-integrated inquiry and guided inquiry. Significant mean gain in procedural knowledge for the conventional approach, however. This finding is similar with previous studies Saha et al. (2010), Abed et al. (2019) and Yimer (2020) which found that student –centered approach enhanced with technology creates learning environment for PSMTs to learn through constructing their own understanding.

6. Conclusion and Suggestions

Although using instructional technology to facilitate learning was advocated as a way to aid in the development of greater understanding, it can be concluded that integrating instructional technology with a guided inquiry-based learning method was responsible for comparably improved conceptual and procedural knowledge. From the outcomes obtained, the following suggestions can be forwarded:

- Mathematics teachers should be proficient on the effective use of computer-assisted instructional approaches, i.e., the integration of technology, pedagogy, and content (TPCK), through seminars, workshops, and conferences, in order to empower their PSMTs in learning geometry.
- Given that guided inquiry teaching techniques supported by technology are capable of enhancing PSMTs' learning outcomes in plane geometry, mathematics educators and instructors should consider these approaches to enhance teaching and learning of the geometric concepts.
- In addition, further empirical studies are needed to develop more precise technological activities to be incorporated into guided inquiry learning strategies in other mathematical areas, such as calculus, algebra, etc.

7. Limitations and Future Research

The study, which was quasi-experimental in design and included three groups of PSMTs, took place in a particular course at two colleges for teacher education. The limitation of this study is the use of a purposive sample for the interventions. Since the sampling is biased, the results cannot be generalized to the entire population of CTEs.

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References

Abed, A. Z., Sameer, S. A., Kasim, M. A., & Othman, A. T. (2019). Predicting effect implementing the jigsaw strategy on the academic achievement of students in mathematics classes. *International Electronic Journal of Mathematics Education*, *15*(1), em0558. https://doi.org/10.29333/iejme/5940

Abiatal, L. K., & Howard, G. R. (2020). Constructivism-led assistive technology: An experiment at a Namibian special primary school. *South African Journal of Childhood Education*, *10*(1), 1-12. https://doi.org/10.4102/sajce.v10i1.794

Adelabu, F. M., Makgato, M., & Ramaligela, M. S. (2019). The importance of dynamic geometry computer software on learners' performance in geometry. *Electronic Journal of E-Learning*, *17*(1), 52-63.

Arbain, N., & Shukor, N. A. (2015). The effects of GeoGebra on students' achievement. *Procedia-Social and Behavioral Sciences*, *172*, 208-214. https://doi.org/10.1016/j.sbspro.2015.01.356

Areaya, S. & Sidelil.A. (2012). Students' difficulties and misconceptions in learning concepts of limit, continuity and derivative. *Ethiopian Journal of Education, 32*(2), 1-37.

Artigue, M., & Baptist, P. (2012). *Inquiry in mathematics education: background resources for implementing inquiry in science and in mathematics at school (Fibonacci project)*. European Union.

Atnafu, M., Teshome, Z. & Kassa, M. (2015). Perception of civil eengineering extension students of Addis Ababa University Institute of Technology in the teaching of applied mathematics. *Ethiopian Journal of Education and Science*, *10*(2), 51-78.

Bokosmaty, S., Mavilidi, M. F., & Paas, F. (2017). Making versus observing manipulations of geometric properties of triangles to learn geometry using dynamic geometry software. *Computers & Education*, *113*, 313-326. https://doi.org/10.1016/j.compedu.2017.06.008

Boopathiraj, C., & Chellamani, K. (2013). Analysis of test items on difficulty level and discrimination index in the test for research in education. *International Journal of Social Science & Interdisciplinary Research*, *2*(2), 189-193.

Bujang, M. A., & Baharum, N. (2017). A simplified guide to determination of sample size requirements for estimating the value of intraclass correlation coefficient: a review. *Archives of Orofacial Science*, *12*(1), 1-11.

Caswell, C. J., & LaBrie, D. J. (2017). Inquiry based learning from the learner's point of view: A teacher candidate's success story. *Journal of Humanistic Mathematics*, *7*(2), 161-186. https://doi.org/10.5642/jhummath.201702.08

Celikten, O., Ipekcioglu, S., Ertepinar, H., & Geban, O. (2012). The effect of the conceptual change oriented instruction through cooperative learning on 4th grade students' understanding of earth and sky concepts. *Science education international*, *23*(1), 84-96.

Cesaria, A. N. N. A., & Herman, T. A. T. A. N. G. (2019). Learning obstacle in geometry. *Journal of engineering science and technology*, *14*(3), 1271-1280.

Charles-Ogan, G. L. A. D. Y. S., & George, N. R. (2015). Investigating difficult concepts in senior secondary school mathematics curriculum as perceived by students. *International Journal of Academic research and reflection*, *3*(6), 67-74.

Clements, D. H., & Sarama, J. (2011). Early childhood teacher education: The case of geometry. *Journal of Mathematics Teacher Education*, *14*, 133-148. https://doi.org/10.1007/s10857-011-9173-0

Cohen, J. (1988). *Statistical power analysis for the behavioral sciences*. Erlbaum.

Cohen, L., Manion, L., & Morrison, K. (2007). *Research methods in education*. Routledge.

Creswell, J. W. (2012). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research*. Pearson.

Dagnew, A., & Mekonnen, D. (2020). Effect of using guided inquiry teaching method in improving grade eight students' concept of photosynthesis, primary school: Ethiopia. *International Journal of Innovative Research in Education*, *7*(1), 1-15. https://doi.org/10.18844/ijire.v7i1.4655

Darkis, J. M. (2020). Views and challenges in teaching mathematics of elementary teachers in rural and urban school districts. *Journal of Critical Reviews*, *7*(4), 107-112. https://doi.org/10.31838/jcr.07.04.19

Donevska-Todorova, A. (2015). Conceptual Understanding of Dot Product of Vectors in a Dynamic Geometry Environment. *Electronic Journal of Mathematics & Technology*, *9*(3), 192-209.

Dorgu, T. E. (2016). Different teaching methods: A panacea for effective curriculum implementation in the classroom. *International Journal of Secondary Education*, *3*(6), 77-87. https://doi.org/10.11648/j.ijsedu.s.2015030601.13

Eshetu, D., Atnafu, M., & Woldemichael, M. (2022). The effectiveness of guided inquiry-based technology integration on pre-service mathematics teachers' understanding of plane geometry. *Journal of Pedagogical Research*, *6*(4), 84-100. https://doi.org/10.33902/JPR.202215241

Evitts, T. (2005). *Investigating the mathematical connections that preservice teachers use and develop while solving problems from reform curricula* [Unpublished doctoral dissertation]. The Pennsylvania State University, USA.

Fantu, M. (2014). *Practice and challenges of practicum at Shambu College of Teacher Education* [Unpublished doctoral dissertation]. Jimma University, Jimma.

Field, A. (2009). *Discovering statistics using SPSS: Book plus code for E version of text*. Sage.

Fraenkel, J. R. & Wallen, N. E. (2006). *How to design and evaluate research in education*. McGraw-Hill.

Francis, O. (2019). *Measures of Students' conceptual understanding and reasoning of upper basic school geometry using two-tier diagnostic tests in Benue State* [Unpublished doctoral dissertation]. Obafemi Awolowo University, Nigeria.

Fujita, T., & Jones, K. (2006). Primary trainee teachers' knowledge of parallelograms. *Proceedings of the British society for Research into learning mathematics*, *26*(2), 25-30.
Fyfe, E. R., McNeil, N. M., & Borjas, S. (2015). Benefits of "concreteness fading" for children's mathematics understanding. *Learning and Instruction*, *35*, 104-120. https://doi.org/10.1016/j.learninstruc.2014.10.004
Gardner, K. (2012). An online community of inquiry for reflective practice in an operative dentistry course. *Journal of Dental Education*, *76*(5), 641-650. https://doi.org/10.1002/j.0022-0337.2012.76.5.tb05298.x
Gemechu, E., Kassa, M., & Atnafu, M. (2018). Matlab supported learning and students' conceptual understanding of functions of two variables: Experiences from Wolkite University. *Bulgarian Journal of Science and Education Policy*, *12*(2), 314-344.
George, D. & Mallery, P. (2003*). SPSS for Windows step by step: A simple guide and reference*. Allyn & Bacon.
Gerard, L. F., Varma, K., Corliss, S. B., & Linn, M. C. (2011). Professional development for technology-enhanced inquiry science. *Review of Educational Research*, *81*(3), 408-448. https://doi.org/10.3102/0034654311415121
Graham, M., Milanowski, A., & Miller, J. (2012). *Measuring and promoting inter-rater agreement of teacher and principal performance ratings.* Center for Educator Compensation and Reform.
Güven, B., & Kosa, T. (2008). The effect of dynamic geometry software on student mathematics teachers' spatial visualization skills. *Turkish Online Journal of Educational Technology-TOJET*, *7*(4), 100-107.
Haapasalo, L. & Kadijevich, D. (2000). Two types of mathematical knowledge and their relation. *Journal fürMathematik-Didaktik, 21*(2), 139–157. https://doi.org/10.1007/BF03338914
Hiebert, J. & Carpenter, T. (1992). Learning and teaching with understanding. In D. Grouws (Ed.), *Handbook of research on mathematics research and teaching* (pp. 65–100). Macmillan.
Hiebert, J., & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.), *Conceptual and procedural knowledge. The case of mathematics* (pp. 1–27). Erlbaum.
Hulme, K. (2013). *The role of technology in the zone of proximal development and the use of van hiele levels as a tool of analysis in a grade 9 module using Geometer's Sketchpad* [Unpublished doctoral dissertation]. University of the Witwatersrand, Johannesburg.
Kandil, S. (2016). *An investigation of the effect of inquiry-based instruction enriched with origami activities on the 7th grade students' reflection symmetry achievement, attitudes towards geometry and self-efficacy in geometry* (Publication no. 439139). [Master's thesis, Middle East Technical University]. Council of Higher Education Thesis Center.
Kandil, S., & Işıksal-Bostan, M. (2019). Effect of inquiry-based instruction enriched with origami activities on achievement, and self-efficacy in geometry. *International Journal of Mathematical Education in Science and Technology*, *50*(4), 557-576. https://doi.org/10.1080/0020739X.2018.1527407
Kasa, M. W. (2015). *Mathematics teacher education and teachers' professional competence in Ethiopia* [Unpublished doctoral dissertation]. Addis Ababa University, Addis Ababa, Ethiopia.
Kilpatrick, J., Swafford, J. & Findell, B. (Eds.). (2001). *Adding it up: helping children learn mathematics*. National Academy Press.
Kothari, C. R. (2004). *Research methodology: Methods and techniques*. New Age International.
Kotu, A., & Weldeyesus, K. M. (2022). Instructional use of Geometer's Sketchpad and students' geometry learning motivation and problem-solving ability. *Eurasia Journal of Mathematics, Science and Technology Education*, *18*(12), em2201. https://doi.org/10.29333/ejmste/12710
Laursen, S. L., & Rasmussen, C. (2019). I on the prize: Inquiry approaches in undergraduate mathematics. *International Journal of Research in Undergraduate Mathematics Education*, *5*, 129-146. https://doi.org/10.1007/s40753-019-00085-6
Lazonder, A. W., & Harmsen, R. (2016). Meta-analysis of inquiry-based learning: Effects of guidance. *Review of educational research*, *86*(3), 681-718. https://doi.org/10.3102/0034654315627366
Leech, N. L., Barrett, K. C., & Morgan, G. A. (2014). *IBM SPSS for intermediate statistics: Use and interpretation*. Routledge. https://doi.org/10.4324/9780203122778
Lewis, D., & Estis, J. (2020). Improving mathematics content mastery and enhancing flexible problem solving through team-based inquiry learning. *Teaching and Learning Inquiry*, *8*(2), 165-183. https://doi.org/10.20343/teachlearninqu.8.2.11
Liao, S. C., Hunt, E. A., & Chen, W. (2010). Comparison between inter-rater reliability and inter-rater agreement in performance assessment. *Annals Academy of Medicine Singapore*, *39*(8), 613. https://doi.org/10.47102/annals-acadmedsg.V39N8p613
Luneta, K. (2015). Understanding students' misconceptions: an analysis of final Grade 12 examination questions in geometry. *Pythagoras*, *36*(1), 1-11. https://doi.org/10.4102/pythagoras.v36i1.261

Magee, P. A., & Flessner, R. (2012). Collaborating to improve inquiry-based teaching in elementary science and mathematics methods courses. *Science Education International*, *23*(4), 353.

Marchis, I. (2012). Preservice primary school teachers' elementary geometry knowledge. *Acta Didactica Napocensia*, *5*(2), 33-40.

Mariquit, T., & Luna, C. (2017). Probing Students' Conceptual Understanding, Mathematical Fluency, and Mathematics Anxiety through Cognitive-Demand Mathematical Tasks. *International Journal of Science and Research (IJSR)*, *6*(8), 988-991.

Meng, C. C., & Sam, L. C. (2013). Developing pre-service teachers' technological pedagogical content knowledge for teaching mathematics with the Geometer's Sketchpad through lesson study. *Journal of Education and Learning*, *2*(1), 1-8. https://doi.org/10.5539/jel.v2n1p1

Ministry of Education [MoE]. (2017). *Education Statistics Annual Abstract 2015/16*. Author.

Ministry of Education [MoE] . (2020). *Education statistics annual abstract September 2019–March 2020*. Author.

Ministry of Education [MoE] . (2012a). *Education Statistics Annual Abstract (2011–2012)*. Author.

Ministry of Education [MoE] . (2015). *Education Sector Development Program IV (ESDP IV).* Author.

Mostert, I., & Clark-Wilson, A. (2016). *Teaching and learning mathematics with technology.* Cambridge University Press.

Murni, V., & Jehadus, E. (2019). Learning circle through geogebra media oriented to understanding concepts. *International Journal of Innovative Science and Research Technology*, *4*(5), 1-4.

National Council of Teachers of Mathematics (NCTM) (2000). *Principles and standards for school mathematics*. Author.

Ndlovu, M., & Mji, A. (2012). Pedagogical implications of students' misconceptions about deductive geometric proof. *Acta Academica*, *44*(3), 175-205.

Ocal, M. F. (2017). The Effect of Geogebra on Students' Conceptual and Procedural Knowledge: The Case of Applications of Derivative. *Higher Education Studies*, *7*(2), 67-78. https://doi.org/10.5539/hes.v7n2p67

Oldknow, A., Taylor, R., & Tetlow, L. (2010). *Teaching mathematics using ICT*. A&C Black.

Pallant, J. (2011). *Survival manual: A step by step guide to data analysis using SPSS*. Allen & Unwin.

Quinn, A. L. (1997). Using dynamic geometry software to teach graph theory: Isomorphic, bipartite, and planar graphs. *The Mathematics Teacher*, *90*(4), 328-332. https://doi.org/10.5951/MT.90.4.0328

Rietveld, T., & van Hout, R. (2015). The t test and beyond: Recommendations for testing the central tendencies of two independent samples in research on speech, language and hearing pathology. *Journal of communication disorders*, *58*, 158-168. https://doi.org/10.1016/j.jcomdis.2015.08.002

Rittle-Johnson, B., Schneider, M., & Star, J. R. (2015). Not a one-way street: Bidirectional relations between procedural and conceptual knowledge of mathematics. *Educational Psychology Review*, *27*, 587-597. https://doi.org/10.1007/s10648-015-9302-x

Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology*, *93*(2), 346-362. https://doi.org/10.1037/0022-0663.93.2.346

Saha, R. A., Ayub, A. F. M., & Tarmizi, R. A. (2010). The effects of GeoGebra on mathematics achievement: enlightening coordinate geometry learning. *Procedia-Social and Behavioral Sciences*, *8*, 686-693. https://doi.org/10.1016/j.sbspro.2010.12.095

Salifu, A. S. (2020). Effect of GeoGebra on pre-service teachers' achievement and perception of circle theorems at E. P. College of Education, Bimbilla-Ghana. *International Journal of Innovative Science and Research Technology*, *5*(8), 1202 – 1215. https://doi.org/10.38124/IJISRT20AUG404

Salifu, A. S., Yakubu, A. R., Ibrahim, F. I., & Amidu, B. (2020). Van Hiele's geometric thinking levels and achievement differences of pre-service teachers' and in-service teachers' in Ghana. *International Journal of Innovative Research and Advanced Studies*, *7*(1), 128-136.

Saunders-Stewart, K. S., Gyles, P. D., & Shore, B. M. (2012). Student outcomes in inquiry instruction: A literature-derived inventory. *Journal of Advanced Academics*, *23*(1), 5-31. https://doi.org/10.1177/1932202X11429860

Sebsibe, A. S., & Feza, N. N. (2019). Assessment of students' conceptual knowledge in limit of functions. *International Electronic Journal of Mathematics Education*, *15*(2), em0574. https://doi.org/10.29333/iejme/6294

Siregar, R., & Siagian, M. D. (2019, October). Mathematical connection ability: Teacher's perception and experience in learning. *Journal of Physics: Conference Series*, *1315*(1), 012041. https://doi.org/10.1088/1742-6596/1315/1/012041

Siyepu, S. W., & Mtonjeni, T. (2014, July). *Geometrical concepts in real-life context: A case of South African traffic road signs* [Paper presentation]. 20th Annual National Congress of the Association for Mathematics Education of South Africa, Kimberley, Northern Cape.

Skemp, R. R. (1978). Relational understanding and instrumental understanding. *Arithmetic Teacher, 26*(3), 9–15. https://doi.org/10.5951/AT.26.3.0009

Star, J. R. (2005). Reconceptualizing procedural knowledge. *Journal for Research in Mathematics Education, 36*(5), 404–411.

Star, J. R., & Stylianides, G. J. (2013). Procedural and conceptual knowledge: Exploring the gap between knowledge type and knowledge quality. *Canadian Journal of Science, Mathematics and Technology Education*, *13*, 169-181. https://doi.org/10.1080/14926156.2013.784828

Tabachnick, B. G., & Fidell, L. S. (2007). *Experimental designs using ANOVA*. Thomson/Brooks/Cole.

Tezer, M., & Cumhur, M. (2017). Mathematics through the 5E instructional model and mathematical modelling: The geometrical objects. *Eurasia Journal of Mathematics, Science and Technology Education*, *13*(8), 4789-4804. https://doi.org/10.12973/eurasia.2017.00965a

Tofel-Grehl, C., & Callahan, C. M. (2014). STEM high school communities: Common and differing features. *Journal of Advanced Academics*, *25*(3), 237-271. https://doi.org/10.1177/1932202X14539156

White, H., & Sabarwal, S. (2014). Quasi-experimental design and methods. *Methodological briefs: impact evaluation*, *8*(2), 1-16.

Wiersma, W., & Jurs, S. G. (2005). *Research methods in education.* Allyn & Bacon.

Wong, W. K., Yin, S. K., Yang, H. H., & Cheng, Y. H. (2011). Using computer-assisted multiple representations in learning geometry proofs. *Journal of Educational Technology & Society*, *14*(3), 43-54.

Yimer, S. T. (2020). Stimulating Content Knowledge Learning of Intermediate Calculus through Active Technology-Based Learning Strategy. *EURASIA Journal of Mathematics, Science and Technology Education*, *16*(12), em1903. https://doi.org/10.29333/ejmste/8705

Yusuf, M. O., & Afolabi, A. O. (2010). Effects of computer assisted instruction (CAI) on secondary school students' performance in Biology. *Turkish Online Journal of Educational Technology-TOJET*, *9*(1), 62-69.

Zulnaidi, H., & Zakaria, E. (2012). The effect of using GeoGebra on conceptual and procedural knowledge of high school mathematics students. *Asian Social Science*, *8*(11), 102. https://doi.org/10.5539/ass.v8n11p102

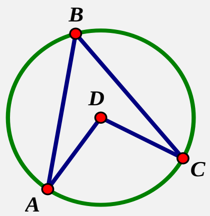
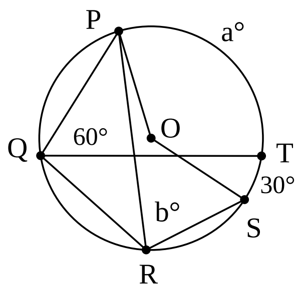
Zulnaidi, H., and Zamri, S. N. A. S. (2017). The effectiveness of the GeoGebra Software: The intermediary role of procedural knowledge on students' conceptual knowledge and their achievement in mathematics. *EURASIA Journal of Mathematics Science and Technology Education*, *13,* 155-2180. https://doi.org/10.12973/eurasia.2017.01219a

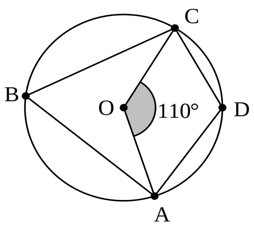
Appendix 1: Sample of 5E lesson Plan

Objective: At the end of the lesson, PSMTs should evaluate the relationship between the central and its inscribed angle.

Prior knowledge: central angle, inscribed angle, chords

Materials: Paper cutter, hard paper, GSP and/or GeoGebra

<table>
<tr><th></th><th>Teacher Activities</th><th>Student Activities</th></tr>
<tr><td>Engage</td><td>Motivate, inspire and arouse curiosity
Determine what students know and believe.</td><td>Listening
Participate answering
Think-pair-share</td></tr>
<tr><td>Explore</td><td>Facilitator and arranges exploration activities.
Guide without direct telling</td><td>What will happen when point A move? When C move? When point B move? What do you conjecture for your exploration? Write your hypothesis. What relationship between the central angle, inscribed angle, the arc AC?
<table>
<tr><th>m∠ADC</th><th>m∠ABC</th></tr>
<tr><td>90.61°</td><td>45.31°</td></tr>
<tr><td>105.52°</td><td>52.76°</td></tr>
<tr><td>108.46°</td><td>54.23°</td></tr>
<tr><td>109.74°</td><td>54.87°</td></tr>
<tr><td>111.35°</td><td>55.67°</td></tr>
<tr><td>97.06°</td><td>48.53°</td></tr>
</table>

</td></tr>
<tr><td>Explain</td><td>Suggest comments</td><td>Explain their hypothesis found</td></tr>
<tr><td>Elaborate</td><td>Asking question to deepen their understanding
Challenging with big hypothesis.</td><td>Apply the concepts in different activities
The relationship between the central and inscribed angle are formulated
Reasonable evidence and conclusion must be conceptualized</td></tr>
<tr><td>Evaluate</td><td>Formal and summative assessment</td><td>1. Compute the following if $P\hat{Q}T = 60^0$, $\widehat{ST} = 30^0$, find the value of arc PT = a^0, $P\hat{R}S = b^0$, $P\hat{O}S =$


2. What is the value of angle $\angle CDA$? $\angle CBA$

</td></tr>
</table>