Journal of Pedagogical Sociology and Psychology
Volume 6, Issue 2, 2024
https://doi.org/10.33902/jpsp.202426544

Research Article

Investigation of mathematics teachers' teaching practices in transition to algebra in the context of letter symbols

Dilara Elbir[1] and Zeynep Medine Özmen[2]

[1]*Damyeri Secondary School, Ministry of National Education, Ordu, Türkiye;* [2]*Trabzon University, Fatih Faculty of Education, Trabzon, Türkiye.*

Correspondence should be addressed to Dilara Elbir dilara_elbir19@trabzon.edu.tr
Received 11 February 2024; Revised 7 April 2024; Accepted 20 April 2024

Algebra is built upon strong arithmetic foundations, so a smooth transition from arithmetic to algebra is crucial. As part of the transition process to algebra, the meaning and use of letter symbols plays a critical role. Meanwhile, the lack of studies with teachers draws attention. Specifically, this study examines secondary school mathematics teachers' instructional practices related to the meaning of letter symbols in the transition from arithmetic to algebra. Through the use of a case study design, the study was conducted in the academic year 2021-2022. The study group consisted of six mathematics teachers from secondary schools. In-class observations and clinical interviews were used as data collection tools. Qualitative analyses were conducted on in-class observations of teachers. According to the findings of the study, the teachers made the transition from informal symbols to letter symbols rapidly, without adequately emphasizing the meanings of the letter symbols. Teachers' practices regarding the meaning of letter symbols should be guided by in-service trainings conducted in this regard.

Keywords: Arithmetic, algebra, transition to algebra, instructional practice, teacher

1. Introduction

The knowledge of arithmetic and algebra allows us to solve a wide variety of problems in our daily lives. The broadest and most applied field of mathematics is arithmetic, which involves performing four operations on known quantities in order to find unknown quantities, and formulating relationships between numbers in order to establish relationships between their values (Akkan, 2009). With the generalization of the relations and rules reached between numbers in arithmetic, the field of learning algebra has emerged (Vance, 1998). According to Kieran (1992), algebra is a tool that reveals number properties and numerical relations, enables the expression of quantities with symbols, and performs operations. At the same time, it is described as a tool for problem-solving and developing abstract thinking, a lesson taught at school (Dede & Argün, 2003), a bridge that provides a conceptual and theoretical transition between mathematics and other disciplines, a prerequisite, and a challenge for further education (Erbaş et al., 2009). In light of the significance of algebra in our lives, it is important that people learn algebra for their own benefit. Van Amerom (2002) states that the foundations of symbolization, generalization, and algebraic thinking are laid together with arithmetic and that the foundations of algebra will be laid with solid learning of arithmetic. Based on this information, it is understood that making sense of arithmetic is the beginning of algebra learning (Watson, 2009). On the other hand, it is stated in the studies that students face difficulties due to the differences between arithmetic and algebra (Akkan, 2009; Gürbüz & Toprak, 2014; Kieran, 2007; Van Amerom, 2002). The most common difficulties experienced by students in the transition from arithmetic to algebra can be listed as the meaning and use of symbols and letters, problem solving, and generalization (Akkan, 2009). It is noteworthy that among these difficulties is the use of letter symbols in problem-solving and generalizing the results (Radford, 2006). Hence the component of making sense of letters forms the basis of the difficulties encountered and is accepted as one of the critical points (Kieran, 1992). Letters are handled with different meanings and in different ways in arithmetic and algebra (Akkan & Baki, 2016a). In order to avoid problems with algebra in the future, it is essential to examine the transition process to algebra and to learn the correct use and meaning of letter symbols. The reason for the cognitive gap that occurs during the transition to algebra is the inability to perform operations on the unknowns. The solution process of arithmetic problems provides meaning and interpretation of the unknown, but it is important to examine how the unknown is related to the variable (Özgeldi, 2013). This transition from arithmetic to algebra will be easier if more than one solution strategy is used in different problem contexts before algebra (Akkan et al., 2012; Van Amerom, 2002). In the solution of the problems in the pre-algebraic process, besides reaching the result with four operations, the use of symbols may be required in cases where four operations are insufficient. Van Amerom (2002) mentions that the development of the pre-algebraic process can be achieved with arithmetic, informal symbolization, and algebraic thinking. The use of informal symbols in problem solving prior to algebra facilitates the transition to algebra by providing the beginnings of abstract thinking (Linchevski, 1995). For this reason, applications for problem solving also play an important role in the transition to algebra. Considering that the reason for the cognitive gap that occurs during the transition to algebra is the inability to perform operations on the unknowns (Özgeldi, 2013), it is understood that the concepts of variable and unknown play an important role in the transition process. Considering the importance of these concepts in algebra, it turns out that it is important to prepare learning environments in such a way that difficulties and mistakes can be avoided (Yıldız et al., 2015). However, it is stated that one of the most effective factors in preventing the difficulties encountered in learning environments and facilitating the transition process of students to algebra is the teacher's knowledge of the subject of arithmetic, algebra, and the transition between these two fields (Akkan et al., 2017). In this sense, it is determined that teachers play an important role in the meaning of letter symbols that play a key role in the transition to algebra. On the other hand, while the difficulties experienced by students, their algebraic understanding and algebraic thinking levels are generally discussed in studies (Akkan & Baki, 2016b; Boulton-Lewis et al., 1997; Çelik & Güneş, 2013; Nayıroğlu, 2022; Ursini & Trigueros, 2011), it has been seen that the studies conducted with the teachers are limited in number (Asquith et al., 2007; Baş et al., 2011; Gürsoy, 2019; Yıldız & Yetkin-Özdemir, 2021). For students to overcome their difficulties in learning algebra, it is important for teachers to practice how to teach the subject. Hence, this study aimed to examine the teaching practices of secondary school mathematics teachers in the transition process from arithmetic to algebra in the context of the meaning and use of letter symbols.

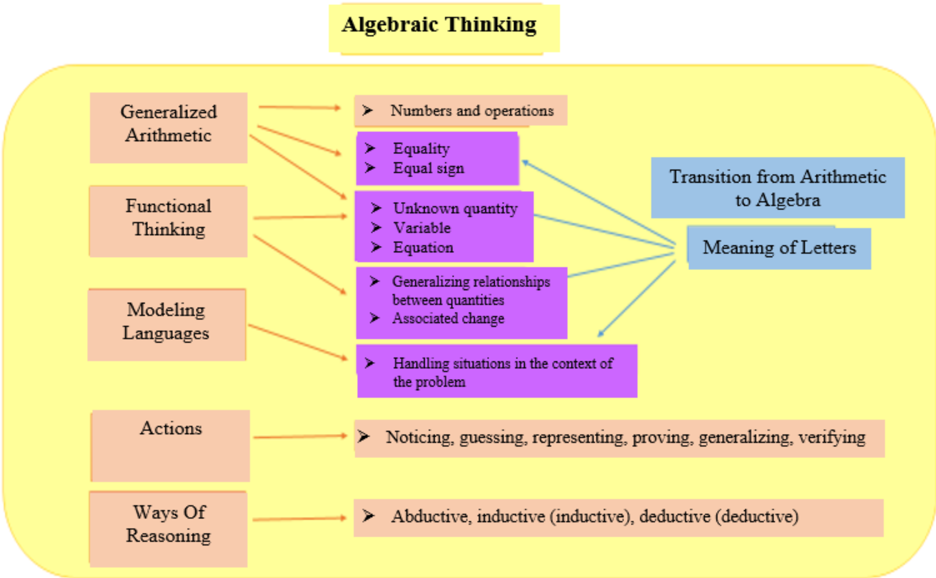
In this context, this research seeks answer to the following question: "What are the instructional practices of secondary school mathematics teachers in the context of the meaning and use of letter symbols in the transition process from arithmetic to algebra?"

2. Theoretical Framework

Kaput (2008) states that algebraic thinking consists of three basic components. These three components are addressed as generalized arithmetic, functional thinking, and modeling languages. In the content of these concepts, it is seen that basic algebra concepts such as equality, equivalence, symbols, variables, unknown quantities, expression are intertwined with the meaning and use of letters in the transition process to algebra. Therefore, the learning and teaching of the meaning and use of letters are also closely related to the development of algebraic thinking. The relationship between algebraic thinking and the meaning and use of letters is illustrated in Figure 1.

Figure 1

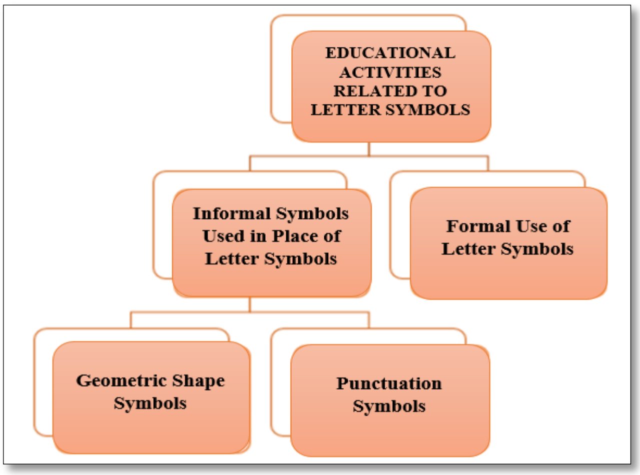
The relationship between algebraic thinking and the meaning and use of letters



Teaching activities related to letter symbols are discussed under two headings in the studies in the literature. Rather than letter symbols, informal symbols used initially. In this process, geometric shapes or punctuation symbols are used to represent unknown or changing quantities informally. The formal use of letter symbols includes representing unknown or changing quantities with letters such as x, y, z, a, b, c... etc. The transition from informal symbols to the use of formal symbols is discussed in the transition from arithmetic to algebra. In this context, this study, which deals with the transition process to algebra, benefits from the classification that is based on the meaning of letter symbols. This classification is given in Figure 2.

Figure 2

The use of letter symbols in the transition from arithmetic to algebra



The use of informal symbols is important in preparing students for algebra and facilitating the transition to algebra (Akkan et al., 2011, 2012). Geometric shapes and punctuation symbols used to represent unknown or changing quantities become less useful over time. For this reason, it is necessary to use of letter symbols rather than informal symbols (Akkan, 2009).

3. Method

Qualitative research method was used in this study, in which the classroom practices of the teachers during the transition to algebra were examined in the context of the meaning of letter symbols. Since this research examined the practices of the teachers in depth with a small number of samples, the case study design has been adopted.

3.1. Participants

The study group of the research consists of 6 secondary school mathematics teachers working in the eastern black sea region of Türkiye. To ensure diversity, the sample was selected considering the criteria of years of experience, holding a bachelor's degree and a master's degree, and working at a village school or district centre. The teachers in the study group were coded as T1, T2, T3, etc. Demographic information of participating teachers is shown in Table 1.

Table 1
Participants's demographic characteristics

Participant	*Professional experience*	*Education status*	*Location of the school*	*Having been trained in algebra*
T1	15	Bachelor's degree	County Town	No
T2	9	Master's degree	County Town	Yes
T3	3	Bachelor's degree	Village School	No
T4	3	Bachelor's degree	Village School	No
T5	9	Master's degree	County Town	Yes
T6	7	Bachelor's degree	County Town	No

3.2. Data Collection Tools

In-class observations and clinical interviews were used as data collection tools in the study. Together with the observations, the observation form prepared in accordance with the purpose of the research was used.

3.2.1. Observation form

To examine the in-class practices of teachers regarding the meaning of letter symbols, an observation form consisting of 15 indicators for these practices was prepared. These indicators have been determined with the help of studies on transition to algebra and algebraic thinking in the literature. Expert opinions were taken into consideration for the determined indicators. In order to determine whether the indicators were suitable for the purpose, a pilot study was conducted. As a result of reviewing the pilot application with experts, a final observation form was designed. These indicators are shown in Table 2.

Table 2
Indicators for the meaning of letter symbols

Code	*Indicators*
G1	Performing numerical calculations without using symbols
G2	Performing arithmetic operations without taking the symbol into account
G3	Drawing attention to the importance of informal symbol use
G4	Preparing the student using informal symbols
G5	Using physical representations or virtual manipulatives for symbol use
G6	Emphasis on comparison of informal symbols and letter symbols
G7	Engaging in activities that the letters represent different dimensions, such as the abundance or quantity of an object
G8	Allowing the use of different symbols
G9	Emphasizing that the letters used can always correspond to a numeric value
G10	Drawing attention to possible misconceptions about the use of letters
G11	Enabling students to discuss different answers about the use of letter symbols
G12	Presenting symbolic and everyday language in a context
G13	Identifying a symbol for varying quantities in a problem situation
G14	Expressing the related quantities with the same symbol in the problem situation
G15	Emphasizing the different meanings of formal uses of letter symbols

Prior to algebra acquisitions, the use of letter symbols was also necessary as part of problem solving activities. This study also examined the teaching practices discussed with the expert opinion regarding the acquisition of problem solving skills. The problem solving acquisition indicators were identified as G1, G2, G3, G4, G13, and G14. In light of these acquisitions, it was decided to choose a 6th grade level. The time spent by teachers on algebra and problem-solving acquisitions is reflected in the observation times. Table 3 shows the durations for observations.

Table 3
Data collection durations

Teachers	*Observation durations (Total=32)*
T1	Problem Solving– 2-hour lesson
	Algebra – 3-hour lesson
T2	Problem Solving – 2-hour lesson
	Algebra – 3-hour lesson
T3	Problem Solving – 2-hour lesson
	Algebra – 3-hour lesson
T4	Problem Solving – 2-hour lesson
	Algebra – 3-hour lesson
T5	Problem Solving – 2-hour lesson
	Algebra – 4-hour lesson
T6	Problem Solving – 2-hour lesson
	Algebra – 4-hour lesson

The algebra acquisitions for which the teachers' practices were examined were as follows:

- Writes an algebraic expression suitable for a verbally given situation and a verbal situation suitable for a given algebraic expression
- Calculates the value of the algebraic expression for different natural number values that the variable can take
- Explains the meaning of simple algebraic expressions
- Solves and constructs problems that require four operations with natural numbers

Considering that the use of informal or letter symbols is also included in the problem-solving acquisitions, it was decided to observe the practices for the last acquisition.

3.2.2. Interview

As part of this study, clinical interviews and classroom observations were conducted to examine secondary school mathematics teachers' practices during the transition to algebra and practices that were difficult for observers to understand. Teachers were interviewed with 11 questions. In order to gather information regarding teachers' algebra knowledge, their explanations of letter symbols, and their plans for teaching algebra, the questions were arranged accordingly. For example, the form included questions like 'What is the variable and the unknown? Can you define? What kind of activities do you do to teach these concepts?' This examination analyzed how they handled questions and concepts. Additional questions were also asked to each teacher about the situations that emerged in his/her own lessons.

The teachers who took part in the study were asked to ask their students questions after they completed their problem-solving tasks. Researchers chose this particular question from a study on the transition to algebra in the literature so that it can be solved using algebraic notation as well as symbols (Akkan et al., 2012). As part of this process, it was important to observe the students' preferences regarding the algebraic method, as well as the solution methods. In order to confirm the results obtained from the classroom practices and see the students' thoughts, the answers of the students were analyzed.

3.3. Data Analysis

Teachers' in-class observations were analysed qualitatively by coding within the scope of the observation form created. The interviews in which the teachers' knowledge about the teaching practices and letter symbols in the transition process from arithmetic to algebra were audio-recorded. Each teacher's lesson was transcribed over the indicators marked on the observation form. Based on the frequencies obtained for each indicator, the percentages were calculated. Transcripts were prepared based on interviews conducted with six teachers using voice recordings. A number of prominent themes were uncovered as a result of coding these data. These findings were presented using direct quotations from data obtained during the observations, accompanied by sample expressions and interviews.

Approximately 30% of the courses were sent to another researcher for validity and reliability studies. The comparisons showed that an 88% consensus was reached.

4. Results

The problem-solving achievement discussed in the transition to algebra is examined by using some of the indicators determined for algebra achievement, so the findings of this section are organized into two main sections, problem solving outcomes and algebra outcomes.

4.1. Results regarding Problem Solving

Table 4 provides the frequencies and percentages of teachers' practices related to the problem-solving achievement based on the indicators.

Table 4 shows that the G1 is the most frequently used indicator with a rate of 58%. In terms of coded indicators, G3 and G4 account for the least coverage (2% each). T5 with a rate of 28.6% had the highest level of practice when the total rates of all the indicators included by the teachers were examined. Compared to this, T4 was the teacher who placed the least emphasis on indicators, with 10.2%. All teachers dealt primarily with the G1 indicator in their problem-solving practices.

Within the scope of the G1 indicator, the problems covered in lessons are generally those that do not require symbols and can be solved using four operations. With a rate of 58%, this is the most frequently used indicator among teachers. Due to the fact that problem-solving performance happens before algebra acquisitions, teachers should focus on informal symbols within this indicator. G13 and G14 indicators, which include informal symbols, are often dealt with by teachers after G1, in this context. Using both informal symbols and four operations, the T5 solved a problem. Figure 3 includes a section of the T5's lesson.

Figure 3

Two different solutions of T5 on the smart board

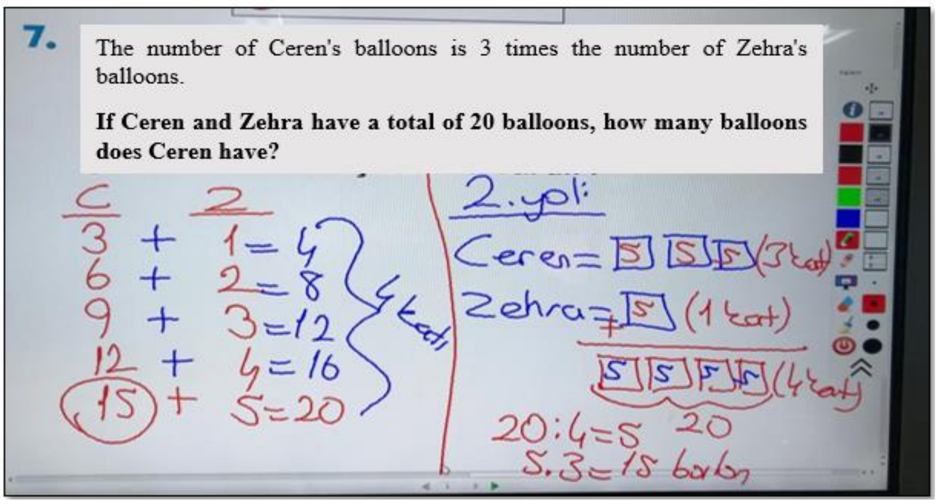


Table 4

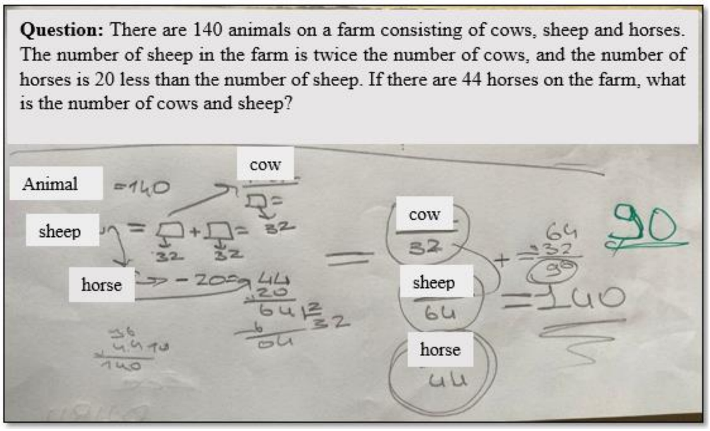
Frequency and percentages of teacher practices regarding problem solving

Code	*Indicators*	*T1*		*T2*		*T3*		*T4*		*T5*		*T6*		*Total*	
		f	%	*f*	%	*f*	%	*f*	%	*f*	%	*f*	%	*f*	%
G1	Performing numerical calculations without using symbols	1	3.4	4	13.8	6	20.7	5	17.2	9	31	3	10.3	28	58
G2	Performing arithmetic operations without taking the symbol into account	3	75	1	25	-	-	-	-	-	-	-	-	4	8
G3	Drawing attention to the importance of informal symbol use	-	-	-	-	-	-	-	-	1	100	-	-	1	2
G4	Preparing the student using informal symbols	-	-	1	100	-	-	-	-	-	-	-	-	1	2
G13	Identifying a symbol for varying quantities in a problem situation	3	37.5	1	12.5	-	-	-	-	2	25	2	25	8	16
G14	Expressing the related quantities with the same symbol in the problem situation	3	37.5	-	-	-	-	-	-	2	25	2	25	7	14
	Total	10	20.4	7	14.3	6	12.2	5	10.2	14	28.6	7	14.3	49	100

As a result of the arithmetic solution, the teacher reminds the students that Ceren has a balloon count of based on Zehra's balloon count. Based on this statement, it is possible to calculate Ceren's balloon number if Zehra's balloon number is known. Furthermore, it emphasizes the relationship between the number of balloons. Using a box symbol, the teacher illustrates the relationship between Zehra's balloon number and the number of balloons. While stating that the problem cannot be solved with arithmetic operations, he makes one feel the need to use a box symbol if larger numbers are involved. Consequently, since the teacher refers to informal symbols for changing quantities, the teacher's statements fall under the G13 category. It was evident from the answers given by the T5's students to the question that they attempted to use informal symbols. Figure 4 shows an example of a student's answer to the question.

Figure 4

Sample answer of a student



Examining the student's answer, it is evident that he represents cows with a box symbol and sheep with the sum of two boxes. The result he found represented the box symbol and indicated the numbers of both sheep and cows.

T1 also indicated that problems including four operations were an important source of students' difficulties. Although he stated this, it seems only one example was used in which the operational process was more weighted in comparison to other examples. An excerpt from this interview can be found below.

> Researcher: Which mathematics subjects and concepts should students know to understand the subject of algebraic expressions? What should be their prior knowledge?
> T1: Of course, you know the most important thing in algebraic expressions is reading comprehension, of course you will process what you understand. That's where you're going to call the unknown x, the student learned that, but they must be able to set up a proper equation, that's the most important thing for us. That's why we need a good reading comprehension skill. And besides, mathematical operations which is indispensable. Negative and positive signs, multiplication and division… the child should be able to address these.

According to the teacher's answer, students should work on problems involving four operations. Therefore, it can be assumed that the teacher understands what students should know as prior knowledge. Although the teacher gives examples that use arithmetic operations in the lesson, he seems to use only one problem that has a painful solution process. Due to the teacher's lack of space for arithmetic operations, students were expected to make solutions by using symbols in the questions given. Based on the solutions given by the students, it was obvious that they were mainly trying to come up with arithmetic solutions.

4.2. Results Regarding Algebra

Table 5 provides the frequencies and percentages of teachers' practices related to the algebraic achievement based on the indicators of letter symbols.

Table 5

Frequencies and percentages of teacher practices regarding algebra

Code	*Indicators*	*T1*		*T2*		*T3*		*T4*		*T5*		*T6*		*Total*	
		f	%	*f*	%	*f*	%	*f*	%	*f*	%	*f*	%	*f*	%
G1	Performing numerical calculations without using symbols	-	-	1	20	1	20	2	40	-	-	1	20	5	3.2
G2	Performing arithmetic operations without taking the symbol into account	-	-	1	100	-	-	-	-	-	-	-	-	1	0.6
G3	Drawing attention to the importance of informal symbol use	-	-	1	50	1	50	-	-	-	-	-	-	2	1.3
G4	Preparing the student using informal symbols	-	-	1	100	-	-	-	-	-	-	-	-	1	0.6
G5	Using physical representations or virtual manipulatives for symbol use	-	-	-	-	4	80	1	20	-	-	-	-	5	3.2
G6	Emphasizing on comparison of informal symbols and letter symbols	-	-	1	25	-	-	1	25	1	25	1	25	4	2.5
G7	Engaging in activities that the letters represent different dimensions, such as the abundance or quantity of an object	-	-	7	29.2	4	16.7	6	25	1	4.2	7	29.2	25	16
G8	Allow the use of different symbols	1	4.3	5	21.7	4	17.4	4	17.4	5	21.7	4	17.4	23	14.6
G9	Emphasizing that the letters used can always correspond to a numeric value	-	-	2	15.4	-	-	2	15.4	5	38.5	4	30.8	13	8.3
G10	Drawing attention to possible misconceptions about the use of letters	1	14.3	2	28.6	1	14.3	2	28.6	1	14.3	-	-	7	4.5
G11	Enabling students to discuss different answers about the use of letter symbols	-	-	-	-	-	-	-	-	-	-	-	-	-	-
G12	Presenting symbolic and everyday language in a context	1	25	1	25	-	-	2	50	-	-	-	-	4	2.5
G13	Identifying a symbol for varying quantities in a problem situation	13	22	10	16.9	8	13.6	12	20.3	4	6.8	12	20.3	59	37.6
G14	Expressing the related quantities with the same symbol in the problem situation	-	-	3	37.5	-	-	2	25	1	12.5	2	25	8	5.1
G15	Emphasizing the different meanings of formal uses of letter symbols	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	Total	16	10.2*	35	22.3*	23	14.6*	34	21.7*	18	11.5*	31	19.7*	157	100

A review of Table 5 shows that all the teachers included the indicator coded with G13 with a rate of 37.6% in algebra outcomes. G13 is the most commonly addressed indicator by T1 (22%). As can be seen, G2 and G4 appear only once (0.6%). It is noteworthy that the teachers did not include practices for the indicator coded with G11 and indicator coded with G15. In terms of percentage, the G7 and G8 indicators followed closely, while the G2, G3, and G4 coded indicators had the least percentage. According to the table, the teachers' practices are largely focused on certain indicators, while the practices related to letter symbols are limited. According to a comparison of the practices by the teachers based on frequency and rate, T2 and T4 demonstrate equal number of applications with a 21.2% ratio. Despite showing 12.1% practice, T1, who covered most of his indicators within the scope of the G13 indicator, included only four indicators. Teachers also handle indicators differently in terms of frequency. In terms of indicator diversity, T1 lags behind the other teachers. Indicator diversity is seen to be the highest in T2's practices.

Only T2 paid attention to the indicator coded with G2. In his response to a question asked in the interview, T2 preferred to use the inverse operations similarly to the way he did in the classroom. As part of the question asked, the teacher used an informal symbol, from which the reverse processing method was used to reach the answer. Consequently, the teacher's solution path in the classroom and in the interview are parallel. A part of the interview with T2 is given below.

> Researcher: In a group of 60 people consisting of women, men and children, the number of women is equal to 3 times the number of men. As there are 12 children in this group, how many boys are there?
> How would you explain the solution of this problem to the students?
> T2: We solve similar questions in natural number problems as well. If the number of women is 3 times the number of men, here is a box for one and 3 boxes for the other. I start with a box and then move on to an algebraic expression. If 12 out of 60 people are children, I will find 48 out of 60 by subtracting 12. One box and 3 more boxes make up 4 boxes. <u>We think that whichever number we multiply by 4 becomes 48</u>. That's how we'll find out.

It is evident from the teacher's explanations that he solved the problem asked in the interview in the same way he solved the questions in the classroom. It is noteworthy that the teacher also uses informal symbols in solving the problem. Accordingly, the observations of the teacher and the data from the interviews are parallel. Rather than addressing it in class, teachers make use of the reverse processing method. According to the clinical interview with T1, the teacher shows a similar solution approach when the same question is asked.

There has been relatively little attention paid to the G5 indicator. In one of the classrooms where this indicator was addressed, T3 made a model by writing the x and y letters on colored cartons. It is also noteworthy that the x and y symbols were written differently. Two symbols representing different sizes are represented by letters with different shapes.

Figure 5

The representations T3 used in the classroom

A close examination of T3's lesson reveals that he uses a concrete material to represent the letters he uses in his lesson. In order to solve the examples, students used these materials. As a physical representation, these concrete materials are considered part of the G5 indicator. Based on an examination of Figure 5, it is evident that the teacher has chosen papers in different colours and shapes representing x, y, and numbers. The following section explains why the T3 teacher used the material in this way.

> Researcher: You used different shapes and colours of paper for x, y and numbers. What is the reason for this?
> T3: I did this to show that the x and y symbols do not represent the same number. I also used it to show whether the x and y symbols can be added or subtracted.

During the clinical interview, it was determined that T3 used models in a variety of colours and shapes to teach similar terms. This situation is also useful for students to understand that the symbols x and y do not represent the same quantity, to enable them to understand why the symbols x and y cannot be summed up or subtracted in addition and subtraction operations in algebraic expressions.

Except for one teacher, all teachers included the G10 indicator in their lessons. As part of his interview, T1 mentions that certain applications were included in order to prevent a certain misconception. Different variables should be used, for instance. When observed, however, these practices are not incorporated into his lessons. An excerpt from this interview is provided below.

> Researcher: What kind of feedback do you give to a student when he comments on a question whose solution is a=8 in the form of "But my teacher came out a=5 in the previous question"? Why do the student thinks this way, in your opinion? What kind of teaching process would you follow to correct this mistake made by the student?
> T1: I told you at first, for example, why did we say x? There may be conceptual misconceptions, such as why this was x, why this happened. That's why the beginning of the topic is very important. <u>We called the variable x, but it could have been y, it could have been z. I think we need to do exercises to ensure that this can be different in any case. That's why it always means x, it means y, I don't think it's right either. That's why I try to give a different variable in each problem as much as I can</u>.

To avoid students from having the misconception that a variable can only be equal to one value, the teacher instructs them to use a different variable in each problem. It was revealed, however, that he used only one variable in all of the problems and solutions he presented in his lectures.

It is noteworthy that G15, an important indicator of the algebra transition, was not covered. At the sixth grade level, the concepts related to understanding letter symbols are variable and unknown when examining the concepts discussed in the transition from arithmetic to algebra. Although letter symbols have meanings other than variable and unknown, these two concepts are emphasized at the secondary school level. What is expected from teachers is to present the concepts of variable and unknown to the students correctly. Consequently, teachers were asked a question about this indicator during interviews. Below is a portion of the T1's interview.

> Researcher: After you introduced algebraic expressions as "expressions containing at least one variable and one operation" in the lesson, you expressed it as "the other name of the variable is also unknown"? Can you explain why you expressed it this way? Is the variable the same as the unknown?
> T1: Yes, I think both concepts <u>have the same meaning</u>. I used the concepts of variable and unknown because I wanted the children to be familiar with both, as both concepts connote the same meaning.

In light of the fact that the teacher taught the concept of variable and unknown as the same, a question was asked to the teacher. A closer examination of the teacher's answer reveals that he believes the two concepts have the same meaning. This shows that the teacher does not understand the difference between these two concepts. Similarly, it was observed that T5 taught the concepts of unknown and variable as the same concept in his lesson. However, it was determined during the interview that T5 understood the differences between the two concepts. As an example, here is a transcript of the interview.

Researcher: What do the variable and unknown mean? Can you define? What kind of activities do you do to teach these concepts?

T5: Variables are expressions that we can give value and that can take different values according to the value we give. We can manipulate variables as we wish. However, the unknown is a single correct value that can satisfy the mathematical equation. We can teach the concept of variable by giving students a pattern situation and asking them to express the relationship according to the number of steps and to find them in the desired different steps based on this relationship.

Researcher: "Or 8a. This algebraic expression means 8 times a. And you can replace a with any number you want. An unknown here. So, it's variable." These were your words in the lesson. Is the unknown also a variable? Can you explain what you mean here?

T5: I used this statement because the expressions "8a" and "8+a" were confused by the students. Of course, the variable and the unknown are not the same thing. However, while a in expression 8a represents the unknown in an equation, it represents the variable in the general term of a number pattern.

A review of the teacher's interview reveals that the teacher understands the difference between a variable and an unknown and explains both concepts in detail. In response to a question about the lesson's situation, he explains that the letter's meaning depends on its location, and he tries to convey this through his expression. In the classroom, however, these concepts may be expressed in a way that may cause students to misunderstand. As a result, it is obvious that the teacher did not emphasize enough this distinction during the lesson.

5. Discussion and Conclusion

According to the findings of the present study, teachers typically referred to G1 as "performing numerical calculations without using symbols" as a problem-solving outcome. According to their practices, teachers prefer problems for which the result can be easily discovered using four operations. Due to the fact that successful transitions to algebra are based on solid arithmetic foundations (Akkan et al., 2011), it may be beneficial to include problem solving in the transition. Due to the importance of arithmetic in the transition to algebra, and the fact that students often struggle with arithmetic operations and four-operation problems, teachers may attempt to improve students' arithmetic skills by emphasizing four operations in algebra problems. This emphasis appears to be effective taking the highest number of G1 indicator into consideration. Algebra problems are often difficult for students to solve (Dede, 2004; Kartal, 2000; Trigueros et al., 2012). The reason for this is students' inability to interpret the problem correctly and their lack of procedural and conceptual knowledge (Dede, 2004; Kartal, 2000; Trigueros et al., 2012). As the G1 indicator was given less room in algebra objectives, it was expected that G2 would be emphasized. The G2 indicator, however, received the least amount of space in algebra objectives. Even though the teachers used symbols to solve problems, they emphasized four operations. According to Akkan et al. (2012), teachers should allow students to solve problems using symbols when transitioning to algebra. Furthermore, it was determined that students should be helped to find their own solutions using symbols.

G13, which refers to 'Determining a symbol for changing quantities in a problem', and G14, which refers to 'Expressing related quantities with the same symbol in a problem', are related each other. Informal symbols should also be used to represent related expressions in the problem. The results also showed that teachers give equal weight to G13 and G14 indicators in their practices. Unlike the other teachers, T5 used symbols as additional ways to solve problems after arithmetic. In this way, students can compare arithmetic and symbolic solutions to the problem. This type of structure is more important to consider as it will help students in their transition to algebra by allowing them to see both solutions. In addition to arithmetic solutions, Akkan et al. (2012) noted that teachers' use of pre-algebraic solution strategies ensure a successful transition from arithmetic to algebra. In this way, studies highlighted to the importance of using different solution strategies (Altun & Sezgin-Memnun, 2008; Arıkan & Ünal, 2012). Furthermore, in their study, Sun-Lin and Chiou (2019) found that students' comparison of algebraic problems and their solutions had a positive effect on their learning. There are studies (Silver et al., 2005; Ziegler & Stern, 2014, 2016) which have similar results. At the same time, it has been mentioned that students should be able to use symbols as a language to explain their thoughts before the transition to algebra (Akkan et al., 2012). During the transition to algebra, teachers should allow students to use informal and lettered symbols to represent certain situations and adopt them. The way teachers addressed indicator G13 in the classroom varied from teacher to teacher, despite the fact that they frequently included it in their practices. After reminding them of informal symbols, some teachers used the letter symbols directly instead of transitioning between them. Additionally, it is recommended that informal symbols be reviewed before moving on to letter symbols. In terms of the outcome of problem-solving, half of the teachers (f=3) recalled informal symbols. According to the other teachers interviewed, informal symbols were used in the 5th grade and letter symbols were switched directly to the 6th. The literature indicates, however, that switching to letter symbols after adequately addressing informal symbols is more effective than teaching algebra traditionally (Nayıroğlu, 2022). On the other hand, Akkan (2009) emphasized that students should have an opportunity to symbolize certain situations with informal symbols before moving on to algebraic concepts. Yıldızhan and Şengül (2017) assert that beginners who encounter algebraic expressions for the first time may be confused by a quick transition without sufficient knowledge of letter symbols. Students may be unable to understand letter symbols if rapid transition instruction is included in practice. Despite expectations that teachers would refer to G13 and G14 equally, in their practices, G14 ranked lower than G13. In algebraic expressions, symbols were used, but they were not applied to quantities related to the expressions. While teachers were expected to refer to G13 and G14 at the same rate, it was observed that they listed G14 with a lower percentage in their practices. It was observed that teachers included the use of symbols in algebraic expressions, but they did not use the same symbol to represent the relevant quantities. A possible explanation is that the focus during algebra transition is on translating verbal expressions into algebraic expressions. Literature defines functional thinking as being able to connect related quantities. However, the literature suggests that the development of functional thinking ensures the transition from arithmetic to algebra (Blanton et al., 2015; Kabael & Tanışlı, 2010; Warren et al., 2006). A quantity is commonly represented with a letter symbol by teachers in this study. Nevertheless, their practices of using the same symbol for related quantities proved to be incomplete. This may result in difficulties in the transition to algebra when limited application of this indicator leaves out functional thinking, which is an integral part of algebraic thinking.

Despite the fact that G7 refers to letters representing dimensions such as multiplicity or quantity of an object, teachers did not include enough of this in their practices. As a result, students were not encouraged to understand what the letter symbols meant. The significance of letter symbols for indicating multiplicity and quantity was not adequately addressed. According to researchers, this situation will lead students to learn letter symbols exclusively for the meaning of objects. According to Akkan and Baki (2016b), students will better understand that numerals are represented by letter symbols if they focus on the multiplicity or quantity of the letters. Teachers should also have practices relating to G7 in this sense. These practices should also address students' misconceptions or misunderstandings about what letter symbols mean. It is frequently mentioned in literature that students have difficulties with letter symbols, and their misunderstandings must be developed in algebra classes (Bozkaya, 2021; Şahin & Soylu, 2011). This is where G10 indicator as pointing out possible misconceptions about the use of letters comes into play. However, it has been observed that teachers do not create a discussion environment for the misconceptions formed during class. Rather than making the student think, teachers give straight explanations. Furthermore, a teacher ignores students' mistakes. Similarly, Şahin and Soylu (2019) found that teachers' teaching practices regarding to overcome misconceptions were not sufficient. In the literature, some studies report similar results (Ball, 1988; Şahin et al., 2016).

In their algebra lessons, teachers failed to emphasize the different meanings of letter symbols according to G15. In order to develop students' algebraic thinking within the G15 indicator, teachers are expected to know the meanings of letter symbols (Akkan, 2009; Uzun, 2021). Uzun (2021) drew attention to the different meanings of letters. Also, studies highlighted that emphasizing the meaning of letter symbols such as variable and unknown in learning environments is an important process for students' algebraic understanding, so teachers should make sure that this is brought to their attention (Asquith et al., 2007; Bardini et al., 2005; Küchemann, 1978; Stacey & MacGregor, 1997). However, teachers seem to use variables and unknowns in the same sense in their lessons. As a result, the teachers' practices were not considered as supporters of the G15.

6. Suggestions

There has been a lack of emphasis placed on the four processes by teachers as well as a lack of consideration given to different solutions strategies. The transition to algebra, however, indicates strong arithmetic foundations. It is therefore proposed to emphasize activities involving four processing skills as part of the transition to algebra. A transition to algebra is more likely to be successful if informal symbols are established first, followed by letter symbols. Teachers should be trained on how to use informal symbols in the transition to algebra through informative in-service training. Teachers should spend more time discussing misconceptions in the literature when teaching algebra because algebra is intrinsically prone to misconceptions. In addition to teachers, it is suggested that teacher candidates should be made more aware by focusing on teaching about misconceptions in the undergraduate education process, and that training should be provided to give feedback to students who have made mistakes or errors during the course. It might be possible to study the effects of incorporating applications for the transition process from arithmetic to algebra in primary school teachers, in line with literature that indicates algebra can be taught at an early age. Studies can be conducted to investigate the effect of teachers' misconceptions in the transition from arithmetic to algebra. Additionally, it was found that teachers with a Master's degree are still unable to understand letter symbols. The meaning of letter symbols should be addressed in algebra courses during graduate education in this direction. Teachers were found to lack knowledge of letter symbols' meanings in the interviews. During undergraduate algebra courses, pre-service teachers should focus on activities that teach different meanings of symbols with more letters.

Acknowledgment: This article was produced from the master's thesis of the first author under the supervision of the second author.

Author contributions: This article consists of a master's thesis. During this process, Zeynep Medine Özmen served as a consultant for Dilara Elbir.

Declaration of interest: No conflict of interest is declared by author.

Ethics statement: Authors declared that the study was approved by the Ethics Committee in Trabzon University on 10.08.2021 with approval code: 2021-10/1.7.

References

Akkan, Y. (2009). *Investigation of the primary school students' transition processes from arithmetic to algebra* (Publication no. 259860) [Doctoral dissertation, Karadeniz Technical University]. Council of Higher Education Thesis Center.

Akkan, Y., & Baki, A. (2016a). Examining the transition processes of secondary school students from arithmetic to algebra: use of symbols and meaning of letters. *Journal of Bayburt Education Faculty*, *11*(2), 270-305. https://doi.org/10.17984/adyuebd.306776

Akkan, Y., & Baki, A. (2016b). Examining secondary school students' transition to algebra in the context of making the properties of the natural number system visible through generalization. *Adıyaman University Journal of Educational Sciences, 6*(2), 198-230. https://doi.org/10.17984/adyuebd.306776

Akkan, Y., Baki, A., & Çakıroğlu, U. (2011). Differences between arithmetic and algebra: the importance of pre-algebra. *Elementary Education Online*, *10*(3), 812-823.

Akkan, Y., Baki, A., & Çakıroğlu, U. (2012). Examining the transition processes of 5-8th grade students from arithmetic to algebra in the context of problem solving. *Hacettepe University Faculty of Education Journal, 43*, 1-13.

Akkan, Y., Akkan, P., & Güven, B. (2017). Awareness of arithmetic and algebra concepts. *Bayburt Faculty of Education Journal, 12*(24), 527-558.

Altun, M., & Sezgin-Memnun, D. (2008). Mathematics teacher trainees' skills and opinions on solving non-routine mathematical problems. *Journal of Theory and Practice in Education, 4*(2), 213-238.

Arıkan, E. E., & Ünal, H. (2012). Problem solving in multiple ways with students with different profiles. *BEU Journal of Science, 1*(2), 76-84.

Asquith, P., Stephens, A. C., Knuth, E. J., & Alibali, M. W. (2007). Middle school mathematics teachers' knowledge of students' understanding of core algebraic concepts: Equal sign and variables. *Mathematical Thinking and Learning, 9*(3), 249–272.

Ball, D. L. (1988). *Knowledge and reasoning in mathematical pedagogy: examining what prospective teachers bring to teacher education* [Unpublished doctoral dissertation]. Michigan State University, Michigan.

Bardini, C., Radford, L., & Sabena, C. (2005). *Struggling with variables, parameters, and indeterminate objects or how to go insane in mathematics* [Paper presentation]. 29th Conference of the International Group for the Psychology of Mathematics Education, Melbourne, Australia, PME.

Baş, S., Çetinkaya, B., & Erbaş, A. K. (2011). Teachers' knowledge about ninth grade students' algebraic thinking structures. *Education and Science, 36*(159), 41-55.

Blanton, M., Stephens, A., Knuth, E., Gardiner, A. M., Isler, I., & Kim, J.S. (2015). The development of children's algebraic thinking: the impact of a comprehensive early algebra intervention in third grade. *Journal for Research in Mathematics Education*, *46*(1), 39-87. https://doi.org/10.5951/jresematheduc.46.1.0039

Boulton-Lewis, G., Cooper, T., Atweh, W., Pillay, H., Wilss, L., & Mutch, S. (1997). The transition from arithmetic to algebra: initial understandings of equals, operations, and variable. In E. Pehkonen (Ed.), *Proceedings of the 21st conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 89–96). PME. https://eprints.qut.edu.au/149409/

Bozkaya, C. (2020). *Investigation of teacher approaches in the transition from arithmetic procedures to algebra* (Publication no. 643623) [Master's thesis, Marmara University]. Council of Higher Education Thesis Center.

Çelik, D., & Güneş, G. (2013). Levels of use and interpretation of letter symbols by students at different grade levels. *Karadeniz Technical University Educational Sciences in Theory and Practice, 13*(2), 1157-1175.

Dede, Y. (2004). Determining the solution strategies used by students when writing algebraic word problems as equations. *Educational Sciences and Practice, 3,* 175-192.

Dede, Y., & Argün, Z. (2003). Why is algebra difficult for students? *Hacettepe University Faculty of Education Journal*, *24*, 180–185.

Erbaş, A. K., Çetinkaya, B., & Ersoy, Y. (2009). Difficulties and misconceptions encountered by students in solving simple linear equations. *Education and Science, 34*(152), 44-59.

Gürbüz, R., & Toprak, Z. (2014). Design, implementation and evaluation of activities that will enable the transition from arithmetic to algebra. *Adıyaman University Necatibey Faculty of Education Electronic Journal of Science and Mathematics Education, 8*(1), 178-203. https://doi.org/10.12973/nefmed.2014.8.1.a8

Gürsoy, P. (2019). *Reflections from a mathematics teacher's algebra teaching process: noticing* (Publication no. 550288) [Master's thesis, Trabzon University]. Council of Higher Education Thesis Center.

Kabael, T., & Tanışlı, D. (2010). Teaching from patterns to functions in algebraic thinking process. *Elementary Education Online, 9*(1), 213-228.

Kaput, J. J. (2008). What is algebra? what is algebraic reasoning? In J. J. Kaput, D. W. Carraher & M. L. Blanton (Eds.), *Algebra in the early grades* (pp. 235–272). Lawrence Erlbaum Associates. https://lccn.loc.gov/2007029532

Kartal, T. (2000). *Evaluation of the high school students' algebraic knowledge in terms of procedural and conceptual knowledge* (Publication no. 96784) [Master's thesis, Karadeniz Technical University]. Council of Higher Education Thesis Center.

Kieran, C. (1992). The learning and teaching of schools algebra. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 390-419). Macmillan.

Kieran, C. (2007). Learning and teaching algebra at the middle school through college Levels. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 707-762). Information Age Publishing.

Küchemann, D. (1981). Algebra. In K. Hart (Ed.), *Children's understanding of mathematics* (pp. 102–119). Murray. https://doi.org/10.1002/9780470996652

Linchevsky, L. (1995). Algebra with numbers and arithmetic with letters: a definition of pre-algebra. *Journal of Mathematical Behavior*, *14*(1), 113-120. https://doi.org/10.1016/0732-3123(95)90026-8

Nayıroğlu, B. (2022). *The effect of the concept of variable in algebra teaching from shape symbols to letter representation on students' achievement and attitude* (Publication no. 715979) [Master's thesis, Fırat University]. Council of Higher Education Thesis Center.

Özgeldi, M. (2013). Interpreting the concepts of unknown and variable in the transition to algebra teaching. *Kastamonu Education Journal, 21*(4), 1317-1326.

Radford, L. (2006). Algebraic thinking and the generalization of patterns: A semiotic perspective. In J. L. C. Alatorre, M. Sa´iz, A. Me´ndez (Eds.), *Proceedings of the 28th conference of the international group for the psychology of mathematics education: Vol. 1* (pp. 2-21). Me´rida.

Şahin, O., Gökkurt, B., & Soylu, Y. (2016). Examining prospective mathematics teachers' pedagogical content knowledge on fractions in terms of students' mistakes. *International Journal of Mathematical Education in Science and Technology*, *47*(4), 531-551. https://doi.org/10.1080/0020739X.2015.1092178

Şahin, O., & Soylu, Y. (2019). Primary school mathematics teacher candidates' instructional strategy knowledge development regarding algebra. *Cumhuriyet International Journal of Education, 8*(3), 887-912. https://doi.org/10.30831/akukeg.335187

Silver, E. A., Ghousseini, H., Gosen, D., Charalambous, C., & Strawhun, B. (2005). Moving from rhetoric to praxis: Issues faced by teachers in having students consider multiple solutions for problems in the mathematics classroom. *Journal of Mathematical Behavior, 24*(3-4), 287-301. https://doi.org/10.1016/j.jmathb.2005.09.009

Stacey, K., & MacGregor, M. (1997). Ideas about symbolism that students bring to algebra. *The Mathematics Teacher*, *90*(2), 110-113. https://doi.org/10.5951/MT.90.2.0110

Sun-Lin, H.-Z., & Chiou, G.-F. (2019). Effects of gamified comparison on sixth graders' algebra word problem solving and learning attitude. *Educational Technology & Society, 22*(1), 120–130. https://www.jstor.org/stable/26558833

Trigueros, M., Ursini, S., & Escandón, C. (2012). Aspects that play an important role in the solution of complex algebraic problems. In T. Y. Tso (Ed.), *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education, Vol. 4* (pp. 147-153). PME. https://d1wqtxts1xzle7.cloudfront.net

Ursini, S., & Trigueros, M. (2011). The role of variable in elementary algebra: An approach through the 3UV model. *Progress in education, 19*, 1-38.

Uzun, N. (2021). *Design, implementation, and evaluation of virtual manipulatives to assist with transition to algebra* (Publication no. 675621) [Doctoral dissertation, Gazi University]. Council of Higher Education Thesis Center.

Van Amerom, B. (2002). *Reinvention of early algebra: developmental research on the transition from arithmetic to algebra* [Unpublished doctoral thesis]. University of Utrecht, The Netherlands.

Vance, J. (1998). Number operations from an algebraic perspective. *Teaching Children Mathematics, 4,* 282-285. https://doi.org/10.5951/TCM.4.5.0282

Warren, E. A., Cooper, T. J., & Lamb, J. T. (2006). Investigating functional thinking in the elementary classroom: foundations of early algebraic reasoning. *The Journal of Mathematical Behavior, 25*(3), 208-223. https://doi.org/10.1016/j.jmathb.2006.09.006

Watson, A. (2009). *Key understandings in mathematics learning-Paper 6: algebraic reasoning.* Nuffield Foundation.

Yıldız, P., Çiftçi, S. K., Şengil-Akar, S. & Sezer, E. (2015). Mistakes made by 7th grade secondary school students in the process of interpreting algebraic expressions and variables. *Hacettepe Journal of Educational Research, 1*(1), 18-35.

Yıldız, P., & Yetkin-Özdemir, I. E. (2021). Teacher subject matter knowledge for the meaningful transition from arithmetic to algebra. *Journal of Pedagogical Research, 5*(4), 172-188. https://doi.org/10.33902/JPR.2021474587

Yıldızhan, B., & Şengül, S. (2017). Examining 6th grade students' misconceptions about the meaning of letters in the context of the transition process from arithmetic to algebra and comparing them with students' mathematics attitudes and self-efficacy. *The Journal of International Lingual, Social and Educational Sciences, 3*(2), 249-268.

Ziegler, E., & Stern, E. (2014). Delayed benefits of learning elementary algebraic transformations through contrasted comparisons. *Learning and Instruction, 33*, 131-146. https://doi.org/10.1016/j.learninstruc.2014.04.006

Ziegler, E., & Stern, E. (2016). Consistent advantages of contrasted comparisons: Algebra learning under direct instruction. *Learning and Instruction, 41*, 41-51. https://doi.org/10.1016/j.learninstruc.2015.09.006