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Research Article

Improving Turkish beginning mathematics teachers' instruction: Using five practices to maintain cognitively demanding tasks

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This qualitative case study investigated the improvement of two beginning middle school mathematics teachers' instruction to implement cognitively demanding tasks and to orchestrate productive discussions by using five practices framework. For this purpose, the study was carried out in two phases: pre and during the professional development (PD). Data were collected through classroom observations, student artifacts, interviews, and teachers' planning and monitoring documents. The results displayed that, before the professional development, teachers did not make plans based on student thinking, not provide adequate time for students to explore the tasks, not build an environment based on classroom discussions, and they usually implemented cognitively low-demanding tasks. Along with the professional development, teachers deeply considered on the purpose of cognitively demanding tasks, but didn't reach an expected level on detailed anticipating. They constructed a classroom setting based on students' exploration of tasks and consideration of multiple solutions. They purposefully selected and sequenced different solutions, initiate discussions to connect students' approaches and underlying concepts. However, they didn't reach the expected level of making connections among different solutions. Eventually, they mostly maintained the cognitive demand of high-level tasks in the professional development period.

Keywords: Five practices, cognitive demand, beginning mathematics teachers, professional development

1. Introduction

Most teachers, who learn innovative approaches during preservice education, tend to use conventional methods after beginning their career (Yanik et al., 2016) because of their initial teaching experiences and challenges (Borko & Putnam, 1996; Lewis, 2014). Although teachers' beliefs and attitudes are affected by many factors, the first few years of teaching profession is crucial for shaping teaching routines (Wang et al., 2008). A report by the Turkish Ministry of National Education Research and Development Department (2008) on the in-service training needs of mathematics teachers emphasizes the importance of PD of 0-5 years experienced teachers. Since the '90s, the need to support beginning teachers has been emphasized in various studies (e.g., Darling-Hammond, 1995; Feiman-Nemser & Parker, 1992;). Thereby, several studies were conducted to support beginning teachers' PD (e.g., Bauml, 2014; Ginns et al., 2001; Harrison et al., 2006). In Turkey, although the majority of studies investigated the challenges of beginning teachers (e.g., Doğan-Coşkun & Işıksal-Bostan, 2018; Taneri & Ok, 2014; Yanik et al., 2016), very few focused on their PD (e.g., Guler et al., 2023).

Since the 90's, expectations from teachers substantially changed. According to National Council of Teachers of Mathematics [NCTM], a teacher's role in the classroom setting should be to encourage students to think, ask questions, solve problems and discuss their ideas, strategies, and solutions (Van de Walle et al., 2013). To create such an athmosphere in the classroom, a teacher should select a worthwhile mathematical task that would ensure students' participation for the intended goal (NCTM, 1991, 2000). Due to this importance, numerous studies examined task quality in instructional materials (Bayazit, 2012; Hong & Choi 2014; Jones & Tarr, 2007; Kotsopoulos et al., 2011), conducted PD studies to improve the quality of implementation and enhance attitudes, mathematical beliefs, content knowledge, and pedagogical content knowledge that affect the implementation of tasks (e.g., Arbaugh & Brown, 2005; Boston & Smith, 2009; Boston & Smith, 2011; Boston, 2013; Clarke et al., 2014; McGraw e al., 2007). However, the number of studies on the experience of implementing mathematical tasks, especially of beginning teachers, is limited. This study aimed to provide it through a specific model named "5 Practices".

1.1. Background

1.1.1. Mathematical tasks

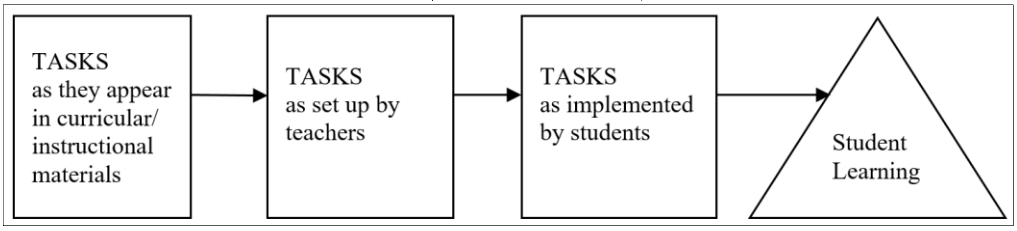
Stein, Smith, Henningsen, and Silver (2000) defined mathematical tasks as a single complex problem or a set of problems aiming to focus students' attention on a significant mathematical idea. Studies (Arbaugh & Brown 2005; Stein & Lane 1996) have shown a deep relationship between types of implemented tasks and students' understanding of key mathematical ideas. Stein et al. (2000) classified mathematical tasks into four categories according to cognitive demand [CD], which is defined as the level of reasoning required to solve a task (see Figure 1).

Figure 1
Indicators of Cognitive Demand of Mathematical Tasks

Degree	Level of CD	Indicators of levels	Example
Low	Memorization (Mm)	Requires recalling previously learned facts, rules, formulas, or definitions.	Memorizing the multiplication table and saying the product of the operation of 5x7.
	Procedures without connections (PwoC)	Includes the implementation of certain procedures without focusing on any meaning.	Finding a product of the operation of "$\frac{2}{3} x \frac{4}{5}$" by using multiplication algorithms such as "multiply the numerator by the numerator, multiply the denominator by the denominator, and write the product."
High	Procedures with connections (PwC)	Aims to understand underlying concepts and ideas and to establish conceptual relations between multiple representations	Focusing on the meaning of the"$\frac{2}{3} x \frac{4}{5}$" multiplication process by thinking about four-fifths of two-thirds on a piece and showing it on a model.
	Doing mathematics (DM)	Requires complex and non-algorithmic thinking and demands students to explore the nature of concepts or processes.	Exploring the underlying algorithm on the multiplication by Napier's Bone technique.

However, selecting a CDT is not sufficient to ensure that it would be implemented at a high level in the classroom as the nature of tasks often changes as they pass from one phase to another (Henningsen & Stein 1997; Stein et al. 2000). Stein et al. (2000) identified four phases entitled mathematical tasks framework [MTF], shown in Figure 2.

Figure 2
Mathematical Tasks Framework (Stein et al., 2000)

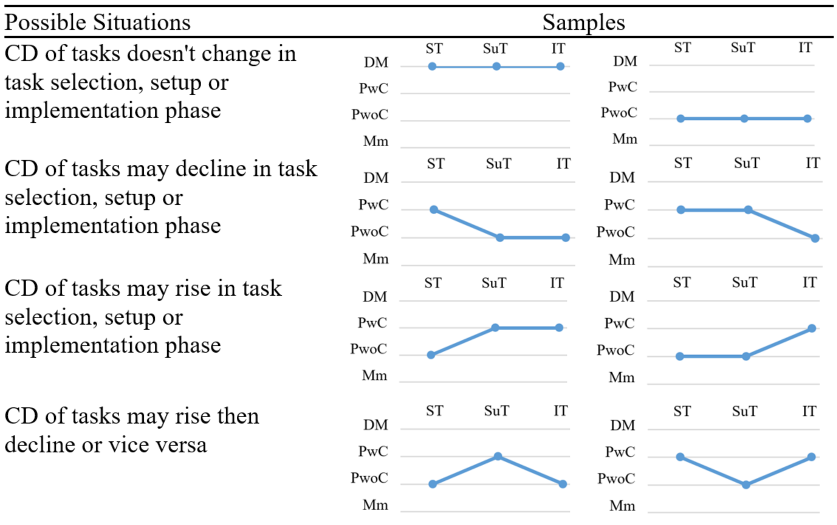


On the other hand, there may be several enactment paths of a task, as shown in Figure 3. Firstly, the CD of a task may not change in the phases of task selection, setup, and implementation.

Secondly, it may decline or increase in one of these stages. Thirdly, it may increase and then decline or vice versa.

Figure 3

Possible Situations in Changes of CD of Tasks



DM: Doing mathematics; PwC: Procedures with Connections; PwoC: Procedures without Connections; Mm: Memorization; ST: Selecting a Task; SuT: Setup of a Task; IT: Implementation of a Task

Studies revealed that not every task provides the same opportunity for students to think and learn (Stein et al., 2000). Learning is closely related to the quality of tasks implemented in the classroom (NCTM, 2014; Stein & Lane, 1996). To implement tasks at a high level, we can build a classroom setting that enables students to think and reason, provide opportunities for establishing conceptual relationships, give enough time to explore tasks, construct the lesson on students' prior knowledge, lead to explain and justify students' ideas. Briefly, we can build a setting to maintain the CD of tasks at a high level. However, studies (e.g., Doğan-Coşkun & Işıksal-Bostan, 2019; Stein et al., 1996; Ubuz & Sarpkaya, 2014) indicated that teachers mostly decline the cognitive demand of mathematical tasks in the implementation phase. Accordingly, it is not realistic to expect beginning teachers to maintain tasks at a high level. Therefore, there is a need for a systematic approach that could support, especially novice, teachers maintaining tasks at a high level. The current study asserts that the notion of 5Ps conceptualized by Smith and Stein (2011) is appropriate for this requirement.

1.1.2. Five practices as a systematic model for PD of beginning teachers

Smith and Stein (2011) designed 5Ps to provide teachers to build classroom understanding based on students' responses. It aims to enable teachers to enact CDTs systematically by setting on certain practices to orchestrate classroom discussions. These practices are briefly anticipating possible solution strategies and misconceptions of CDTs, monitoring students' explorations, purposefully selecting and sequencing multiple solution strategies to discuss, and connecting students' solution strategies and key mathematical ideas, as seen in Table 1 for a detailed explanation.

There are several PD studies that reached successful improvements on teachers' using the 5Ps framework. For instance, Pang (2016) implemented 5Ps in the Korean context with a lesson study design. The results of the study showed that there was a significant change in determining learning objectives, selecting CDTs carefully and meaningfully, designing the classroom setting to maximize the participation of students, sharing ideas, and whole-class discussions. Heyd-Metzuyanim et al. (2018) used Sfard's (2008) notion of a "ritual towards exploration" to theorize the learning trajectories of two secondary school teachers participating in a PD designed within the

Table 1
Five practices and its detailed explanation

Practice	*Explanation of practice*
Anticipating	Teacher set the goal(s) of a lesson, determine the key mathematical ideas, and select an appropriate task. Then, the teacher anticipates the solution strategies of students, either correct or incorrect, and considers how they can be associated with the concepts, representations, and operations. Being aware of possible solutions and misconceptions, the teacher anticipates how to respond to solutions during the monitoring phase. Finally, the teacher anticipate which of these solutions to select, in which order to sequence, and how to help students to make connections between the solutions and the goal(s) of the lesson.
Monitoring	Teacher monitors the students' exploration, asks questions to reveal students' strategies, and note down the solution strategies that may be appropriate for whole-class discussion.
Selecting-sequencing	Teacher purposefully selects particular solution strategies and sequences them to be shared in the classroom. He can use various sequencing strategies such as incorrect solutions to correct ones, frequent solutions to infrequent ones, commonly used representations to less-used representations, concrete to abstract solution strategies, and incomplete solutions to complete ones. This order may be as it was anticipated in planning, or it may be arranged during the lesson by the emergence of solutions that could not be anticipated previously.
Connecting	Tacher helps students to establish a relationship between the solution strategies and to connect the key mathematical ideas of the lesson. The lesson aims to discover key mathematical ideas by connecting multiple solution strategies instead of reaching the answer by presenting different solutions separately (Smith & Stein, 2011).

framework of "5Ps" and "Accountable Talk." The study displayed two teachers' movement from imitative, rigid, and internally inconsistent engagement to a more explorative way. On the other hand, in a study conducted with prospective teachers (Tyminski et al., 2014), although students experienced difficulties in making connections, they showed significant developments.

Although 5Ps have a huge potential on teachers' PD, there haven't been adequate research outcomes for beginning teachers.This study is based on the hypothesis that teachers would be able to maintain CDTs by systematical enactment of 5Ps. Hence, we investigated task selection, implementation, and routine practices (if any practices of 5Ps were observed) of teachers before the PD to reveal teachers' practices in all aspects to set an effective PD. The purpose of this study was to examine the impact of a 5Ps-based PD on the classroom practices and CDs of enacted tasks of two beginning middle school mathematics teachers.

2. Method

This holistic multiple-case study (Yin, 2003) investigates the "change" deeply in the planning and implementation practices of teachers. Yin stated that case studies are based on "how" and "why" questions, and Merriam and Tisdell (2015), on the other hand, underlined the detailed description of the cases in conducted research.

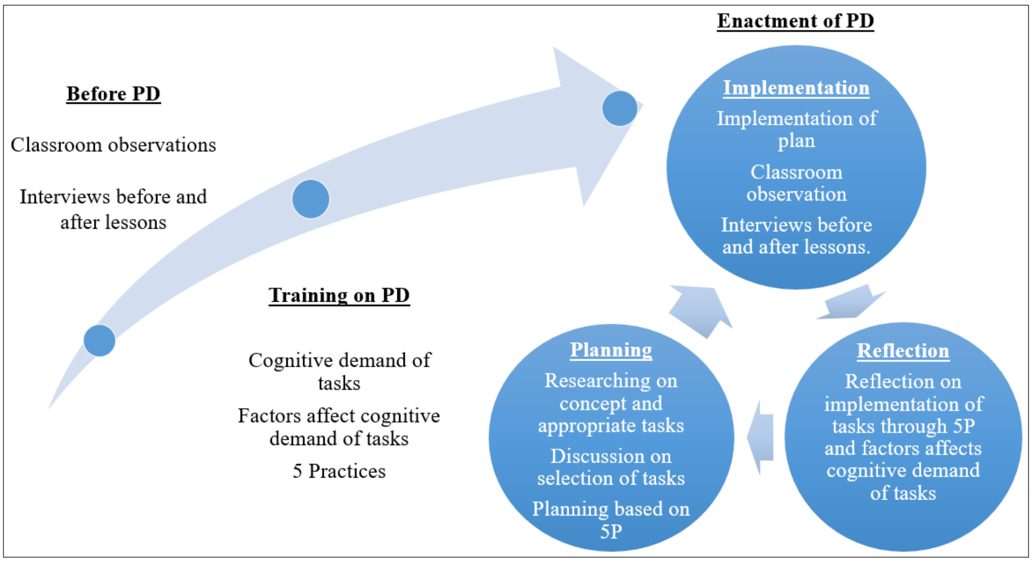
2.1. Context and Participants

The current study was conducted with two volunteer middle school mathematics teachers named Gizem and Duru. Both of the teachers work in a rural district, socioeconomically low-status public schools. Gizem's school is 40 km away from the city center. The biggest problems of her school are low academic achievement, parents' irrelevance, and absenteeism. Her classrooms consist of a maximum of 15 students each, and only a few of them actively participate in lessons. On the other hand, Duru's school is 65 km away from the city center. Although her classrooms has a maximum of 30 students each, her students' academic achievement is higher than Gizem's students. Both teachers graduated from the same faculty and were classmates at university, so it facilitated in communication and collaboration of the participants and researchers. Furthermore, both of them had been sustaining their graduate studies in a mathematics education master program.

2.2. Professional Development Setting and Data Collection

This study was designed in two phases as pre-PD and during PD, as shown in Figure 4.

Figure 4
Research Phases



As mentioned before, this study aims to examine the effects of the 5Ps-based PD on teachers' classroom practices and implementation of tasks. To reveal the effect of the PD, Duru's and Gizem's lessons were observed before the PD for two months, respectively, for 11 hours and 10 hours, regardless of specific concepts and grades. These observations aimed to reveal the extent of the teachers' practices based on the 5Ps framework and the change of CDs of tasks in the phases of task selection, setup, and implementation before the PD.

Following year, the teachers participated in a year-long PD. In the first two months of PD, teachers participated 16 hours (4 modules) of training that consists of sorting the CD of tasks, being aware of the factors that cause the maintenance or decline of the CD, and comprehending the 5Ps. The content of the training is in Table 2.

As seen in Table 2 the training mostly focused on mathematical tasks, but only the last session was about 5Ps. Because, we firstly considered that teachers should comprehend mathematical tasks framework very well to understand 5Ps. Secondly, we already conducted the rest of the PD as a practical intervention that includes iterative cycles of planning, implementation, and reflection based on 5Ps throughout the school year. While I participated in all PD stages as a facilitator, the second researcher mostly supported me to prepare the content of theoretical training, determining the following meeting's topic by examining the video clips together and coding issues. Due to teachers' schedules and preferences, each of them chose only one 6th grade to conduct the study and we observed these classrooms. I and the teachers came together seven times to reflect on their instructions and did planning based on 5Ps. Before these meetings, they researched the next lesson's key concepts and considered on the task they would be planning.

During the planning phase, we discussed to determine the tasks to be implemented in their classrooms, and then teachers filled in the planning form based on anticipating component of 5Ps, which was adapted from "Thinking through lesson protocol" (Smith et al., 2008). Duru and Gizem's lessons were observed, respectively, for a total of 18 and 20 hours, and they both implemented nine CDTs in a year-long period. The goals and tasks are shown in Table 3.

Table 2
Goals of training modules

Modules	*Goals of the modules*
Module 1	What is a mathematical task? What is the cognitive demand? Examining the indicators of cognitive demand levels of mathematical tasks Determining the cognitive demand levels of mathematical tasks Making a connection between the skills anticipated in the mathematics curriculum and cognitively demanding tasks
Module 2	What is the mathematical tasks framework? Examining different cases from studies (Smith, Silver, & Stein, 2005a, 2005b, 2005c) and comparing learning opportunities of students in these cases Identifying the factors that cause to decline and maintain the cognitive demand of tasks and examining these factors through sample dialogues and videos
Module 3	Modifying mathematical tasks (What can be done to increase the quality of mathematical tasks, to increase the cognitive demand level of the tasks?) Designing cognitively demanding tasks
Module 4	5 Practices as a framework to maintain cognitively demanding tasks Teacher and student roles during implementation of 5 Practices Making sample plans to anticipate students' solution strategies Identifying strategies to select and sequence student solutions Making whole class discussion to connect goals, ideas, strategies

Table 3
The tasks implemented in professional development

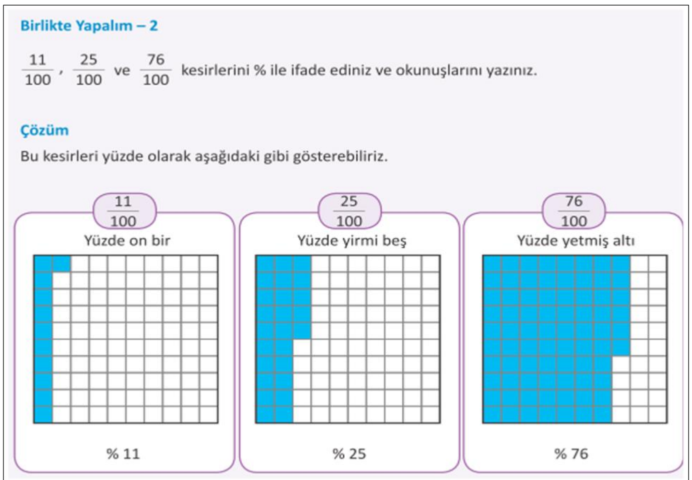
Tasks	*Goal of Tasks*
Lemonade Task [LT]	Exploring the proportional relationship between quantities
Division of Fractions Task [DFT] Milk Task [MT]	Comprehending the division of fractions and exploring division algorithm
Air Temperature Task [ATT]	Comprehending the magnitude of integers and place them in a number line
Sea Coast Task [SCT]	Associating the concept of absolute value with distance
Monthly Expenditure Task [MET]	Using the model and number line to make sense of addition in integers
Tiling a Patio Task [TPT]	Exploring the algebraic relationship in patterns.
Garden Fence Task [GFT]	Associating operations in algebraic expressions with daily life
Triangle Task [TT]	Exploring the area of the triangle by using known planar shapes

We videotaped the enacted lesson, audiotaped all conversations between teachers and students while launching tasks, monitoring solution strategies, or connecting ideas, and photographed all artifacts, whether they include correct or incorrect solutions. Besides, we did semi-constructed interviews before and after the implementation of tasks. These interviews included questions aiming to reveal teachers' thoughts on how to implement tasks before the lesson and how these tasks were implemented after the lesson. After the implementation, we purposefully selected appropriate sections from the classroom videos implemented by teachers before and during PD to be discussed in the reflection phase. These sections were about situations in which teachers routinely decline or maintain the CD of tasks before and throughout PD or challenges and progressions in implementing 5Ps' subcomponents during this period. In the reflection phase, we examined video recording sections, student artifacts, and selecting-sequencing forms (if they used during implementation), discussed on just mentioned challenges and progressions, then teachers completed planning document after the reflection phase. This cycle was followed throughout a school year.

2.3. Data Analysis

In this study, changes in the CD of tasks during the phases of selecting a task, setup, and implementation were coded through the Mathematical Tasks Analysis Framework (Stein et al., 2000) and the levels of using 5P subcomponents were coded through the 5Ps Analysis Framework. We identified the phases from selecting to implementing a task as an analysis unit. If a task includes sub-items that are follow-ups of each other, we evaluated them as only one task. However, if the sub-items of a task are independent, we considered them as separate tasks. The CD of a task may decline, rise, or maintain in task selection, setup, or implementation phase. For instance, the solution of the sample task (see Figure 5) observed in Duru's classroom before the PD based on connecting the percentages with the 10x10 table representation. Therefore, we coded it as a level of PwC. However, Duru changed the structure of the task during the setup phase. She asked the first part of the task to the students and dismissed the connection approach used in the textbook (establishing a relationship with the 10x10 table). Therefore, we coded this task as declining from PwC to PwoC in the setup phase.

Figure 5
A Task that Duru Declined the CD in the Setup Phase (Ministry of National Education [MoNE], 2014, p. 319)



Translation of task: Let's Do Together-2: Express following fractions as percents, $\frac{11}{100}, \frac{25}{100}, \frac{76}{100}$, and write their readings. **Solution:** We can show these fractions in percent as follows.

In the current study, we classified 5Ps components under four headings: 1) planning (task selection, goal setting, and anticipating), 2) monitoring, 3) selecting-sequencing, and 4) connecting. We formed an analysis framework based on previous research (Eskelson, 2013; Smith & Stein, 2011) and identified subcomponents to determine the extent to which teachers were able to implement 5Ps. We scored each of these subcomponents as 0, 1, or 2 points. Table 4 indicates these subcomponents and indicators.

The planning subcomponents were coded by analyzing teachers' statements in planning documents, meetings, and pre and post-lesson interview transcripts. The other subcomponents were coded by analyzing first researcher's observation notes, pre and post-lesson interview transcripts, enacted lessons' and meetings' video and audio recording transcripts, selecting-sequencing forms (if) filled by teachers. To give an example of the coding of a goal-setting subcomponent; for instance, in a task related to the division operation of fractions more than one key mathematical idea can be put forward, such as 1) understanding that the division involves partitioning or measuring, 2) understanding that the quotient may be higher than dividend

Table 4
Subcomponents of five practices

Score	Indicators of planning subcomponents
	Setting a goal
0	The key mathematical idea underlying the task wasn't identified.
1	A single key mathematical idea underlying the task was identified.
2	Multiple key mathematical ideas underlying the task were identified.
	Anticipating possible solution strategies
0	No solution strategy was anticipated.
1	A single multiple solution strategy was anticipated.
2	Multiple solution strategies were anticipated.
	Anticipating possible misconceptions
0	No error or misconception was anticipated.
1	A single error or misconception was anticipated.
2	Multiple errors or misconceptions were anticipated.
	Responding to possible solution strategies
0	No anticipation made how to respond to solutions.
1	Anticipated how to respond to a single solution.
2	Anticipated how to respond to multiple solutions.
	Sequencing solution strategies in the planning
0	No sequencing made in the planning.
1	Solution strategies were randomly sequenced.
2	Solution strategies were purposefully sequenced.
Score	Indicators of monitoring subcomponents
	Duration of monitoring
0	No time was given to students to explore the task.
1	Less or more time was given to students to explore the task.
2	Adequate time was given to students to explore the task.
	Scaffolding and questioning
0	Provided too much information, and there was a lack of questioning.
1	Provided too much information, or there was a lack of questioning.
2	Provided enough scaffolding and questioning.
	Noting down solution strategies
0	Solution strategies were not noted down.
1	Solution strategies partially noted down.
2	Solution strategies noted down.
	Social interactions
0	The students individually explored tasks.
1	The students partially explored tasks with pairs or with larger groups.
2	The students all explored tasks with pairs or with larger groups.
Score	Indicators of selecting and sequencing subcomponents
	Selecting solution strategies purposefully.
0	No solution strategy was selected.
1	A single solution strategy was selected purposefully, or multiple solution strategies were selected randomly.
2	Multiple solution strategies were purposefully selected.
	Sequencing solution strategies purposefully.
0	Solution strategies were not sequenced.
1	Solution strategies were randomly sequenced.
2	Solution strategies were purposefully sequenced.

Table 4 continued

Score	Indicators of connecting subcomponents
	Making connections between solution strategies and the goal(s) of the lesson
0	No discussion occurred, or no connection made between solution strategies and the goals of the lesson.
1	During the discussion, a connection made between a single solution strategy and the goals of the lesson.
2	During the discussion, connections made between multiple solution strategies and the goals of the lesson.
	Making connections among solution strategies
0	No discussion occurred, or no connection made among solution strategies.
1	During the discussion, a connection made between two solution strategies.
2	During the discussion, connections made among multiple solution strategies.

3) exploring the division algorithm. We coded as 2 points for this subcomponent if more than one of these goals is stated, 1 point if one of these goals is stated, and 0 points if no goal is stated by teachers in the planning document or transcripts.

Finally, we calculated the mean score of each subcomponent of the 5Ps to reveal the overall performance of teachers, divided the range of 0-2 points into three equal segments (see Table 5), and classified the following ranges (0 - 0.67) is low, [0.67 - 1.33) is partial, and [1.33 - 2] is high.

Table 5

The mean score of subcomponents of five practices

Level	Low	Partial	High
Score Interval	(0 – 0.67)	[0.67 – 1.33)	[1.33 – 2]

We carried out coding of CD levels and subcomponents of 5Ps together as researchers. We separately coded CD levels of 96 tasks, 78 of which were implemented before the PD and 18 during PD, and found 91.8% coder reliability. We then discussed disagreed tasks until reaching a consensus. The coding procedure of the sub-components of the 5Ps was much more comprehensive because, for every subcomponent, we examined lesson plans, selecting-sequencing forms, lesson videos, meeting documents, and student artifacts. As researchers, we coded 52 subcomponents of the first two tasks together out of 2 x 9 x 13 = 234 subcomponents. During this period, we mostly focused on development of the rubric and eventually reorganized scoring indicators of some sub-components. After reaching an agreement on the indicators of subcomponents', the rest of the tasks coded by the first researcher.

3. Findings

The findings are presented under two headings as teachers' classroom practices before PD and changes in teachers' classroom practices during the PD.

3.1. Teachers' Classroom Practices before Professional Development

In the current study, pre-PD observations and interviews showed us that both of the teachers preferred teacher-centered instructions. They didn't prefer multiple approaches and focused on the consolidation of formulas rather than the meaning behind concepts. Because of her beliefs about her students' low capacity, Gizem usually aims to reinforce concepts by solving a lot of routine examples, without digging up complex parts of the problems. In one excerpt, Gizem stated that "*I usually invite every student on the board to solve plenty of examples*." In another one, "*I'm talking too much, but I'm sure that they couldn't get what all I said. I repeat by supposing that if someone missed, maybe he/she would comprehend it in the second or in the third time.*" Compared to Gizem, Duru was giving more time for students to explore tasks. However, she was careful about sustaining the lesson as she planned in her mind; in other words, she didn't provide opportunities to try different strategies or methods. The following excerpt from PD may confirm our statement: "*In the PD* *meeting, I watched a part of my lesson before the PD, which badly impressed me. I noticed that I had been cutting students' voices a lot, and I wasn't giving any opportunity to express their ideas."*

Observations and interviews conducted before the PD showed that teachers did planning based on selecting exercises rather than the possible student strategies related to the tasks. Before the PD, teachers allowed time to explore some tasks. As shown in Table 6, Duru gave time in %58 of tasks and spent 2.30 minutes per task on average. On the other hand, Gizem gave time in %24 of tasks and spent 1.17 minutes per task on average. During these waiting times, teachers quickly checked student solutions and gave evaluative feedback on whether their solution strategies are correct or incorrect rather than monitoring their solution strategies to give formative feedback. Moreover, any group work was observed before the PD.

Table 6
Duration of time given students during the professional development

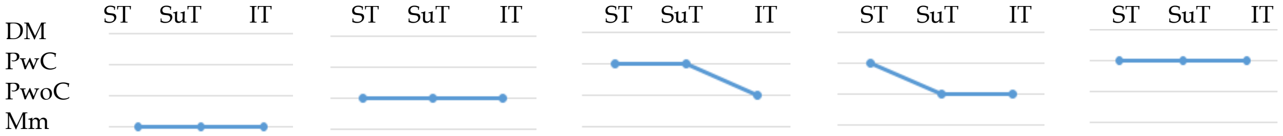
	The time given tasks/ All tasks	The total duration of time given tasks/time given tasks	The total duration of time given tasks / All tasks
Duru	26 / 45 = % 58	113 dk / 26 ≈ 4.20 dk	113 dk / 45 ≈ 2.30 dk
Gizem	8 / 33 = %24	42.30 dk / 8 ≈ 5.19 dk	42.30 dk / 33 ≈ 1.17 dk

While Duru and Gizem selected students to share their solutions, respectively, in 53% and 39% of tasks, both teachers focused on a single correct solution in all of these tasks. Students were usually invited to the board to show the correct solution of a task. Therefore, no situation was observed, such as purposefully selecting and sequencing different solutions or misconceptions. Finally, there were hardly ever classroom discussions based on students' solution strategies. They mostly focused on only one single solution strategy and made no connection between solution strategies.

3.2. Cognitive Demand of Tasks before the Professional Development

Table 7 shows the changes in the CD of tasks in task selection, setup, and implementation phases before the PD. These findings indicate that Duru selected more CDTs than Gizem, but she wasn't able to maintain CD of tasks at a high level. On the other hand, while Gizem selected fewer CDTs, she was mostly able to maintain the CD of tasks at a high level. As a result, Duru and Gizem, respectively, were able to implement only 20% and 21% of tasks at a high level. Furthermore, none of the teachers selected a task at a DM level.

Table 7
The cognitive demand of tasks before the professional development



Duru	3 (%7)	23 (%51)	7 (%15)	3 (%7)	9 (%20)
Gizem	2 (%6)	22 (%67)	2 (%6)	-	7 (%21)

Note. DM: Doing mathematics; PwC: Procedures with Connections; PwoC: Procedures without connections; Mm: Memorization; ST: Selecting a Task; SuT: Setup of a Task; IT: Implementation of a Task

Of the 45 tasks implemented by Duru, 3 of them were at the level of Mm, 23 tasks were at PwoC, and 19 tasks were at PwC level. However, Duru declined 3 out of 19 PwC tasks into PwoC in the setup phase and 7 out of 19 PwC tasks into PwoC level in the implementation phase. On the other hand, of the 45 tasks implemented by Gizem, 2 of them were at the level of Mm, 22 tasks were at PwoC, and 9 tasks were at PwC level. However, Gizem declined 2 out of 9 PwC tasks into PwoC in the implementation phase and maintained 7 tasks at a high level.

3.3. Changes in Classroom Practices of Teachers during the Professional Development

In the current study, both teachers implemented nine tasks during a PD. We have examined the extent to which teachers put in the practice the subcomponents of 5Ps. Table 8 shows the progression of teachers in all of these subcomponents. As shown in Table 8, Duru has a higher score (1.29) than Gizem (1.19) in the overall average. The subcomponents for which teachers scored the lowest were responding possible solution strategies, sequencing solution strategies on planning, noting down solution strategies, and making connections among solution strategies. On the other hand, both of the teachers scored high for the subcomponents of goal setting, anticipating solution strategies, duration of monitoring, scaffolding and questioning, selecting and sequencing strategies purposefully. Duru also scored high in the subcomponent of making connections between solution strategies and the goal(s) of a lesson.

When Gizem's scores were examined in detail, she particularly improved her scores for the subcomponents of the duration of monitoring and scaffolding and questioning. She constructed a better social environment compared to Duru. Besides, while she wasn't able to provide an athmosphere for making connections between solution strategies and the goal of a lesson at the beginning of the PD, she was able to provide it for at least one purpose within the process. Duru provided adequate monitoring time in all tasks from the beginning of the PD, constructed a setting based on questioning, purposefully selected, and sequenced multiple solution strategies in all tasks. How these quantitative findings emerged qualitatively during the PD are explained in detail in the further sections.

3.4. Planning

Before the PD, we observed that both teachers did planning to select questions to be solved in the classroom rather than thinking about student ideas, possible solutions, and misconceptions that may arise. In the meetings held together within the PD, we aimed to be made plans based on 5Ps. Thus, we provided them resources to be read, led them to think especially on the goals and cognitive demand of tasks and fill out the lesson planning forms. We asked them pay particular attention to select CDTs involving multiple representations and multiple solutions that deepen students' conceptual understanding. However, they sometimes had challenges in setting goals despite examining sources. Considering their lack of content knowledge, we made extra lessons on certain topics. For example, one of the most challenging subjects for teachers in setting goals was expressing shape patterns algebraically in the Tiling A Patio Task, so we provided a sample lesson video to facilitate their goal setting. Thus they determined multiple goals for the lesson. Furthermore, the discussions at the meetings enabled teachers to set critical goals that they had not considered before. For example, they thought that the position of "0" in the Air Temperature Task and the Sea Coast Task might be a challenge for students. Therefore a goal should be set to determine the position of 0 in the number line, and it should be discussed in a whole-class discussion. As seen in Table 8, both of the teachers set multiple goals in many of the tasks. For example, in the Division of Fractions and Milk Tasks, they set goals, such as understanding that the division involves partitioning or measuring, realizing situations where the quotient may be higher than the dividend, and discovering the division algorithm.

Before the PD there was no anticipation regarding any task, but along with PD, we asked teachers to think about the possible solutions and misconceptions that may arise for designed tasks. Moreover, we expected them to solve some of these tasks in detail, beyond detailed anticipating, to consider which solutions would be discussed and how to connect these solution strategies in an actual classroom setting. Teachers were able to express their ideas more comfortably and consider multiple solutions, especially on subjects they had sufficient content knowledge and that they did detailed research. While they were able to anticipate multiple solution strategies in most of the tasks, they couldn't reach the desired attainment in the subcomponent of anticipating misconceptions. For example, In the Air Temperature Task, in

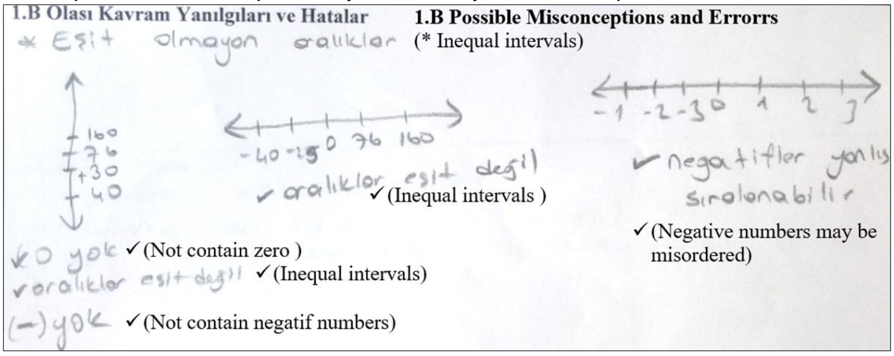
Table 8
Teachers' scores on five practices subcomponents

		Planning					*Monitoring*				*Selecting and Sequencing*		*Connecting*		
	Task	Setting a goal	Anticipating possible solution strategies	Anticipating possible misconceptions	Responding to possible solutions strategies	Sequencing solution strategies on planning	Duration of monitoring	Scaffolding and questioning	Noting down solution strategies	Social interactions	Selecting solution strategies purposefully	Sequencing solution strategies purposefully	Making connections between solution strategies and the goal of a lesson	Making connections among different solution strategies	MEAN
Duru	LT	1	2	1	1	1	2	2	0	1	2	2	1	0	1.23
	DFT	2	2	2	0	0	2	2	1	1	2	2	2	1	1.46
	MT	2	2	0	0	0	2	2	2	2	2	2	1	0	1.31
	ATT	2	2	2	1	2	2	2	1	1	2	2	2	0	1.62
	SCT	2	2	1	1	0	2	2	0	1	2	2	2	1	1.38
	MET	2	2	0	0	0	2	2	0	1	2	2	1	0	1.08
	TPT	2	2	1	0	2	2	2	0	1	2	2	2	2	1.54
	GFT	1	1	0	0	0	2	2	0	1	2	2	1	0	0.92
	TT	1	2	0	0	0	2	2	0	2	2	2	1	0	1.08
	MEAN	1.66	1.89	0.77	0.33	0.55	2	2	0.44	1.22	2	2	1.44	0.44	1.29
Gizem	LT	1	2	1	1	1	1	1	0	1	2	2	0	0	1.00
	DFT	2	2	2	0	0	2	1	0	1	1	0	1	0	0.92
	MT	2	2	0	0	0	2	1	0	1	2	1	0	0	0.85
	ATT	2	2	2	1	2	2	2	2	2	2	2	2	1	1.85
	SCT	2	2	1	1	0	2	2	0	1	2	2	2	1	1.38
	MET	2	2	0	0	0	2	2	0	1	2	2	1	0	1.08
	TPT	2	2	1	0	2	2	2	2	2	2	2	2	0	1.62
	GFT	1	1	1	0	0	2	2	0	1	2	1	1	0	0.92
	TT	2	2	0	0	0	2	2	0	2	2	2	1	0	1.16
	MEAN	1.77	1.89	0.88	0.33	0.55	1.88	1.66	0.44	1.33	1.88	1.55	1.11	0.22	1.19

which teachers anticipated multiple solution strategies and misconceptions successfully, they anticipated that students would use representations to place air temperatures such as thermometer, and vertical and horizontal number lines. Furthermore, as shown in Figure 6, they anticipated that students might not pay attention to the distances between big numbers (e.g. +55, −40) when putting them on the number line, that they might have challenges on the place of "0", and they might make mistakes when putting negative numbers on the number line.

Figure 6

Anticipated Misconceptions of Teachers for Air Temperature Task



In the PD, teachers were expected not only to anticipate possible solutions and possible misconceptions but also to consider how to respond to these solutions and how to sequence the solutions if they occur in an actual setting. However, they didn't consider how to respond to students' multiple solution strategies in any of the tasks, but they considered how to respond to a single solution strategy for only three tasks. Furthermore, they purposefully sequenced different solution strategies in only two tasks. For instance, in Air Temperature Task, in which teachers consider how to responded to students' single solution strategy and purposefully sequenced solution strategies, teachers anticipated that students would have difficulty as which of the numbers −3.5 and −3.75 are greater. They considered that it might be possible to respond by asking such a question, "which is closer to −3 and which is closer to −4". They also sequenced solution strategies to the planning document as 1) solutions, including misconceptions, 2) solutions, including the vertical number line, 3) solutions, including the horizontal number line, to make a connection with the integer concept.

3.5. Monitoring

In the current study, we hadn't observed any subcomponents of monitoring in teachers' classrooms before the PD. Along with the PD, we tried to improve their monitoring skills by getting them to watch their classroom videos and discussing efficient follow-up questions. These meetings provided essential changes in the sub-components of monitoring during PD. Table 9 shows the times given to the students in the monitoring phase by Duru and Gizem.

Table 9

Monitoring times given by teachers during professional development

Task	*LT*	*DFT*	*MT*	*ATT*	*SCT*	*MET*	*TT*	*GFT*	*TPT*	*MEAN*
Duru	42.00	30.10	13.20	40.05	37.10	24.30	27.55	25.25	32.50	30.22
Gizem	33.25	29.05	26.45	36.40	21.20	26.20	47.35	44.50	33.00	34.20

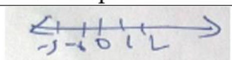
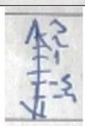
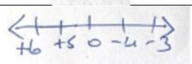
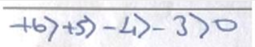
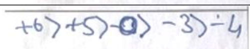
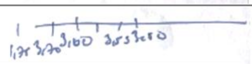
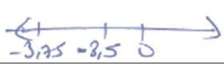
Although Duru and Gizem, respectively, spent 30.22 and 34.20 minutes on average for monitoring per task, time differences in the duration of the monitoring changed according to the subjects and task context required for the students to explore. For instance, in the Lemonade Task, Gizem explained the task to all students one by one by keeping the monitoring time longer than necessary. This situation caused the monitoring time to be too long and, thus, to accelerate the connecting phase. While Duru implemented all subcomponents of the Lemonade Task, Gizem wasn't able to complete the task, and she got 1 point because of keeping time longer than necessary. Gizem stated that "*If the students do not understand during the monitoring, they will have challenges in making connection phase*." Therefore, she frequently emphasized her students' low academic achievement and considered monitoring as a practice where she would teach one-on-one rather than a process where she provides students to explore tasks and produce strategies. However, after watching students' reactions in Duru's video clips in the first meeting, she was more careful in arranging the monitoring time. Thereby, the teachers carried out adequate monitoring time in the remaining tasks.

Although teachers gave students time to explore the tasks in all tasks, it took time to build an environment specific to monitoring and change in students' habits. In the pre-PD observations and at the beginning of the PD, students tended to complete their solutions quickly and ask the teacher whether their answers were correct in both teachers' classrooms. During the meetings, we provided them to watch video clips of some pre-PD lessons and made discussions on teachers' and students' behaviors. This approach had positive results in Duru's classroom from the very beginning. As it can be observed from the scores she got, she avoided giving excessive feedbacks and used explorative questioning to understand students' ideas. Unlike Duru, Gizem gave excessive feedbacks to the students at the beginning of the PD and prompted them to the correct answers. However, examining the video sections of teachers' classrooms led her to provide a setting for more productive questioning.

We discussed the advantages of noting down solution strategies, introduced the selecting-sequencing form (Smith and Stein, 2011), and did some exercises about it. For example, in some cases, we examined all student strategies and selected ones by the teacher to compare and discuss solutions that the teacher didn't choose but could be important, whether correct or incorrect. However, as can be seen from the scores of the teachers, they weren't able to develop the norm of noting down solution strategies regularly. Gizem explained why she could not use it with the following sentences *"I sometimes noted down solution strategies but sometimes I forgot it on my table. Frankly, it takes a process to get used to maintain it but I couldn't do it."* Figure 7 shows an example of a selecting-sequencing form prepared for the Air Temperature Task, which is one of the two tasks in which she scored 2 points.

Figure 7

Gizem's Sample Selecting-Sequencing Form

Student	Solution path	Order
S1		3
S2		1
S3		2
S4		4
S5		5
S6		6
S7		7

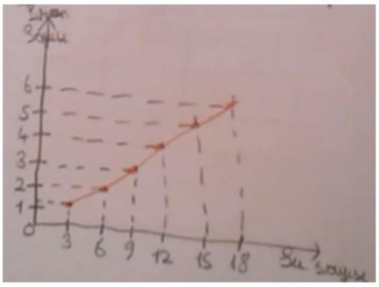
Gizem selected solutions that contain horizontal-vertical lines and correct-incorrect solutions. She sequenced them as beginning with incorrect vertical and horizontal lines and continuing with correct sequenced solutions.

Finally, we agreed with teachers that student collaboration would help discuss ideas and develop new solution strategies. However, even though some students in both classes worked with their groupmates, some resisted collaboration throughout the PD.

3.6. Selecting and Sequencing

While a discussion-based classroom setting was not very common before PD, we aimed with the PD that teachers focus mainly on students' multiple solution strategies and make connections based on student thinking. The cycle of planning-implementation-reflection has provided successful improvements on teachers since the first meeting. We reminded them that selecting a CDT does not guarantee the emergence of different solutions, so they should build a setting to gain the habits of using multiple representations and trying multiple solutions. In the first meetings, teachers argued that students were not used to different representations and solutions. Therefore, during the monitoring phase, they purposefully encouraged students to use different representations and solution strategies by asking questions such as "*Can you find a different solution strategy?*", "*Can you do it by using the number line?*", "*Can you show it by using a model?*". Besides, we argued that encouraging students who completed the task early to a second solution strategy would prevent classroom management problems and provide different solutions. As can be seen from Figure 8, Duru selected multiple solution strategies for all tasks, and purposefully sequenced them all. On the other hand, Gizem didn't select multiple solutions for only one task, and she didn't make a purposeful sequence for two tasks.

Figure 8
Selected Solution Strategies of Duru in the Lemonade Task

<table>
<tr><th>Order</th><th>Selected solutions</th></tr>
<tr><td>1</td><td><table><tr><td>Lemon</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>Bottle of water</td><td>3</td><td>6</td><td>9</td><td>12</td><td>15</td></tr></table></td></tr>
<tr><td>2</td><td><table><tr><td>Lemon</td><td>4</td><td>4 + 1 = 5</td></tr><tr><td>Bottle of water</td><td>12</td><td>12 + 3 = 15</td></tr></table></td></tr>
<tr><td>3</td><td>
</td></tr>
</table>

For instance, in the Lemonade Task where the ratio of $\frac{1}{3}$ for $\frac{\text{lemon}}{\text{water}}$ should be explored, teachers anticipated that students might use table representations, make additive reasoning such as $\frac{1+1+1}{3+3+3}$, or even sketch graphs if successful students were encouraged. They also anticipated that several students might have misconceptions such as $\frac{12}{4} = \frac{12+1}{4+1} = \frac{13}{5} = \frac{13+1}{5+1} = \frac{14}{6}$. As a result, different solutions occurred during the lesson, as expected. Duru selected multiple solution strategies, as shown in Figure 8, and purposefully sequenced them. She firstly selected the solution of Student 1, which illustrates the additive relationship via a table representation. She then selected the solution of Student 2, which denotes the multiplicative relationship via a table. He found $\frac{1}{3}$ by simplifying the fraction of $\frac{4}{12}$ and found five by adding 1 to 4 and fifteen by adding 3 to 12. Duru then selected the solution of Student 3 by saying, "*Let's invite Student 3, who first put forward the idea of graphics.*" In Gizem's classroom, students used only the table representation, and fewer solution strategies emerged compared to Duru's classroom. Gizem firstly selected a solution involving a misconception and then three different solution strategies. Figure 9 illustrates the solutions selected by Gizem.

Figure 9
Solution Strategies for Lemonade Task in Gizem's Class

Order	Selected solutions				
1	5 lemon	13 bottle of water			
	6 lemon	14 bottle of water			
	7 lemon	15 bottle of water			
2	3	6	9	12	15
	1	2	3	4	5
3	4	?			
	12	15			

Gizem purposefully sequenced these solutions with beginning a strategy involves a common misconception in the classroom, then the table representation that illustrates the additive relationship, and finally, a rarely occurred solution that represents the multiplicative relationship.

3.7. Connecting

Teachers' pre-PD classroom practices showed us that while Gizem didn't allow almost any discussion, there were rare discussions in Duru's classroom. Therefore, in the meetings held in the first weeks of the PD, we expected teachers to evaluate their and students' roles by getting them to watch their video clips from the pre-PD classrooms. In these sections, we focused on teacher-student and student-student interactions, teacher questioning and feedback, initiating discussion, and students' participation. When teachers watched these video clips, they noticed that they generally did not allow students to express their ideas. Duru said, "*I was very impressed by my classroom videos. I noticed that I was interfering a lot and didn't allow students to explain their ideas. The student on the board wasn't writing without hearing my command.*" Since the first week of the PD, there had been a significant change in Duru's lessons. Although Duru hadn't yet orchestrated discussions purposefully at the beginning of PD, she provided a classroom setting based on student ideas. Scoring at least 1 point from each task for the subcomponent of making connections between solution strategies and the goals of the lesson shows that she discussed student's ideas in all of the lessons. On the other hand, the fact that Gizem's insistence on teacher-centered approach, especially at the beginning of the PD, prevented her from constructing a discussion-based environment. Gizem's approach in the first few tasks was generally "*show and tell the solutions one by one,*" but watching clips affected Gizem. However, particularly since the Air Temperature Task and the Sea Coast Task involving integers, absolute values, and the operations of integers, she was able to make connections between the goals of the lessons but unable to make connections among solution strategies. In Sea Coast Task, both teachers scored 2 points from the subcomponent of making connections between the goals of the lesson. The following dialogue is from the Sea Coast Task in Gizem's class:

[The fish is 30 meters below from sea level, the coral is 110 meters below from sea level, the bird is 110 meters above sea level, and the kite is 80 meters above sea level (which are in the same direction).]

T: [She invited Student 1 to the board and asked] Where are the fish and coral?
[He subtracted 30 from 110]
T: S1, why did you subtract?
S1: I subtracted to find the distance between them.
T: We just subtracted, should we always subtract? We added to find the distance between the bird [+110 meters] and the fish [−30m].
S2: We add up when both are positive, and we subtract when both are negative.

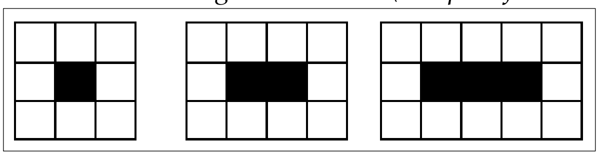
T: Well, sometimes you added up, but sometimes subtracted numbers. They were correct, but you didn't tell me why. Now, what's the distance between the bird and the fish?
S4: 140 m.
T: We found 140 and verified it by counting distance. Then you subtracted +110 from +80 to find the distance between the kite and the bird. And here, you subtracted −30 from −110 and got +80. The results are correct; you have already verified it by counting on the number line.
S5: We add up if both are negative, and subtract if both are positive.
T: Did we do it in negative numbers? We subtracted −20 from −120. S6, what is your opinion?
S6: If both are positive, we subtract. If one is negative and one is positive, we add up.
T: Why?
S7: It asked to find the distance between the bird and the kite.
S6: We add up if one is negative, and one is positive. If both are negative or both are positive, we subtract.

Before the PD, we observed that teachers mostly aimed to reinforce routine algorithms rather than underlying meaning. However, throughout the PD, as seen from the dialogue above, they tried to initiate discussions leading students to make connections between the goals of the lessons and to generalize the ideas behind routine algorithms. These discussions, which are more frequent in Duru's classroom, allowed students to connect the goal(s) of the lessons. Gizem expressed her ideas about the development of questioning skills and students' participation with the following sentences:

"Compared to last year, there are lots of changes both in them and in me. For example, students listening to each other's ideas or a student's reflection on another student's generalization are completely learned behaviors."

Making connections between solution strategies was one of the most difficult subcomponents for teachers. As can be seen from Table 8, both of the teachers made connections between two solution strategies for only two tasks, and Duru, additionally, made connections among more than two solution strategies in a Tiling a Patio Task, which adapted from Smith and Stein's (2011) research. The Tiling a Patio task, as shown in Figure 10, asks the numbers of white tiles in an unknown step.

Figure 10
A Pattern in Tiling a Patio Task (Adapted from Smith and Stein, 2011)



The following dialogue shows an exemplary classroom discussion on making a connection between solutions for the Tiling a Patio Task.

T: Your friends found [2n+6], [2.(n+2)+2], and [3.(n+2)-n]. [All three solutions are on the board]. Well, do you all see the same pattern? For example, what did you find in the 3rd step for each solution?
S1: 12.
T: For the 10th step?
S2: 26.
T: Well, our algebraic expressions seem a little different; we got 2n+6, 2.(n+2)+2, and 3.(n+2)-n. Why are they different? For example, let's write the 200th step for each pattern.
S1: 200 plus 2 equal to 202. 202 times 2 equals to 404, and 404 plus 2 equals to 406.
T: Do they all equal to 406? They look like different algebraic expressions. Are they all the same?
S1: Yes.
T: But why are these algebraic expressions different? Think about it at home. It is your homework.

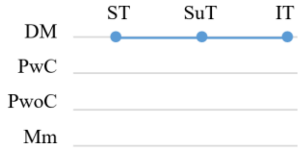
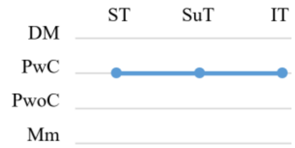
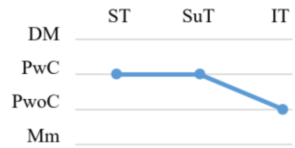
In this dialogue, Duru discussed three different solutions of a task. She led students to discover that different solution paths yield the same results by making connections between these solution strategies. Thus, she scored 2 points for the subcomponent of making connections among solution strategies.

3.8. Changes in the Cognitive Demand of Tasks during the Professional Development

Before the PD, teachers mostly had selected low demanding tasks or had declined the CD of tasks in the implementation phase. However, during the PD, they selected CDTs and implemented them at a high level, except for a task in which Gizem declined its CD, as shown in Table 10.

Table 10

Changes in the cognitive demand of tasks during the professional development

	Implemented at a high level				Implemented at a low level	
						
	Duru	Gizem	Duru	Gizem	Duru	Gizem
LT			✓			✓
DFT			✓	✓		
MT			✓	✓		
ATT			✓	✓		
SCT			✓	✓		
MET			✓	✓		
TPT	✓	✓				
GFT			✓	✓		
TT			✓	✓		

The teachers implemented almost all of the tasks at a high level. Because teachers mostly provided sufficient time for students to explore the tasks, used different representations such as tables, models, graphics, number lines, and they encouraged students to justify their ideas. However, Gizem gave excessive feedbacks and provided students too much time to explore the Lemonade Task, which is the reason why she declined the level of the CD to the PwoC level. In the Tiling a Patio Task, which is the only task implemented at the level of DM, students revealed three different mathematical models by analyzing the relationship between the steps of a pattern, reached a generalization, and explained their strategies.

4. Discussion and Conclusion

The purpose of this study was to examine the impact of a 5Ps-based PD on the classroom practices and CDs of enacted tasks of two beginning middle school mathematics teachers. The findings of this study showed that five practices based intervention provided significant changes in the classroom routines of teachers that weren't observed before the PD. They maintained the level of CDTs at a high-level in practice. While successfully implemented some subcomponents of 5Ps, they had difficulties in implementing some of the others.

Studies indicate that while the CDs of mathematical tasks included in the classroom resources are mostly high (Kotsopoulos et al. 2011; Ubuz et al., 2010), teachers were not able to maintain the level of CDTs in the implementation phase (Doğan-Coşkun & Işıksal-Bostan, 2019; Henningsen & Stein, 1997; Stein et al., 2000; Ubuz & Sarpkaya, 2014). The findings before the PD, similarly, indicated that teachers were only able to implement 20% of the tasks at a high level and had typical classroom routines. Some of these routines can be listed as not giving students enough time to explore the task, routinizing the problematic part of the task, focusing on only one correct answer, giving excessive feedbacks and prompting students to the correct answer. Along with the PD, teachers almost managed to maintain CD of all tasks at a high level and both of them implemented one task at the level of DM that they had never experienced before. There are several studies aiming awareness regarding the CD of tasks and the implementation of CDTs at a high level (e.g. Arbaugh & Brown 2005; Boston & Smith 2009). These studies are in line with the results of this study and showed that PDs provided improvement in task implementation at a high level. Enacting CDTs at a high level requires engaging process skills (MoNE, 2013; NCTM, 2000) into the classroom setting. Stein and Henningsen (1997) and Wilson et al. (2015) stated that in such a setting, students reason on a problem, discuss and connect their ideas, make an effort to find different solutions, use multiple representations, shortly create a student-centered learning athmosphere. In the current study and many of the studies (e.g. Clarke et al. 2014; Stein & Lane, 1996), similar classroom norms were observed where CDTs were enacted and maintained at a high level.

In the current study, significant changes were observed in the practices of teachers, along with the PD. The meetings held during the planning enabled teachers to think more in-depth concerning the selection of CDTs and determining the goal(s) of the tasks. In particular, they paid attention to consider key mathematical ideas underlying the tasks and a structure where students can reason and come up with multiple solutions. Although teachers did not have challenges in setting goals and planning subjects in which their content knowledge (CK) and pedagogical content knowledge (PCK) were sufficient (e.g. area of triangle and integers), they demanded the supported of the researchers in the subjects they had difficulties (e.g. absolute value and algebra). Therefore, as Wilson et al. (2015) stated that PDs lead teachers to a research-based setting; our teachers also researched the determined subjects with the support of the researchers. Although this study did not have a direct goal to enhance teachers' CK and PCK, we observed that a PD focused on improving classroom practices not integrated with CK and PCK would not be sufficient to enhance teachers' practices.

Collaborative meetings contributed to the development of teachers, in particular, teachers' reflections on each other's classroom videos and discussion of cases have led to significant changes in their perceptions, beliefs, and subsequent practices. Borko et al. (2008) and McGraw et al. (2007) stated that using video clips to improve the quality of classroom discussions and reflecting on videos of their courses provided teachers to see their improvement, and watching colleagues' videos helped them learn new pedagogical strategies.

In the planning phase, the teachers got achievement in setting the goal(s) of the lesson, selecting a task, and anticipating possible solution strategies. However, they did not reach the desired level in subcomponents of anticipating possible misconceptions, responding possible solution strategies, and sequencing solution strategies that require detailed anticipation. Eskelson (2013) also found a similar result in his study that teachers could not go beyond writing the objectives identified in the curriculum while preparing a lesson plan and did not make detailed anticipation. On the other hand, in a few studies, teachers made progress (Pang 2016; Silver et al., 2005; Wilson et al. 2015) concerning detailed anticipation. In this study, it can be asserted that the teachers' lack of PCK based on student thinking prevents them from making sufficient anticipation.

One of the most crucial practices was monitoring phase. As Wilson et al. (2015) stated, the monitoring phase focuses on students' solution strategies beyond "answers." Although one of the teachers had difficulty at the beginning of the PD, both teachers generally took care of productive questioning that encouraged students to think about different solution strategies, and they avoided giving excessive feedbacks to prompt the answers. As one teacher in Larsson's study (2015) stated, "It's quite hard, but it's extremely important that you don't tell if it's right or wrong because then you have removed what's the problem in the problem (p. 102)". Wilson et al. (2015) remarked that focusing on misconceptions and multiple solutions for tasks in planning enabled teachers to carry on successful monitoring. Unlike our study, Pang (2016) indicated that one of the most challenging parts of teachers' practice was monitoring and that teachers could not realize the changes in student solutions. In the current study, teachers were unable to transform the subcomponent of "noting down student solutions," which Smith and Stein (2011) emphasized, into a classroom norm, except for one or two tasks. On the contrary, in the study of Heyd-Metzuyanim et al. (2018), teachers conducted successful monitoring by noting down student solutions.

During the PD, teachers purposefully selected and sequenced solution strategies in most of the tasks. They followed numerous sequencing strategies, such as those from erroneous solutions or misconceptions to correct solution strategies, frequent solutions to infrequent ones, commonly used representations to less-used representations, concrete to abstract solution strategies, and incomplete solutions to complete solutions (Larsson, 2015; Meikle, 2014). In the study of Silver et al. (2005), teachers stated that sharing erroneous solutions may have the potential to confuse students; however, these debates can reveal important misconceptions and are pedagogically valuable. Besides, Boaler and Humpreys (2005) and Larsson (2015) indicated that it is an influential approach for students to correct their erroneous solutions on the board themselves. Such an approach has also appeared in some cases in the current study.

In this study, one of the teachers created a discussion-based classroom athmosphere based on student solution strategies from the very beginning of the PD. The other one insisted on emphasizing mathematical procedures with a teacher-centered approach, particularly at the beginning of the PD; nevertheless, she succeeded in creating a discussion-based classroom setting along with PD. Teachers were often able to create an atmosphere of discussion to connect at least one goal of the lesson. Tyminski et al. (2014) found a similar result that the prospective teachers achieved 75%-80% success in setting specific goals for students to make connections among strategies made choices that could support these goals. Pang (2016), on the other hand, indicated that the most challenging practice of teachers was the connection phase. Eskelson (2013) also concluded in his study that teachers were unable to make connections because they did not make detailed anticipation in planning. In the current study, one of the main reasons for teachers' success in making connections with the goal(s) of the lesson is setting the goal(s) of the lesson clearly during the planning and then implementing CDTs on these goal(s).

One of the fundamental practices that Smith and Stein (2011) emphasized is making connections among different solutions or representations. They stated that detailed anticipation is a prerequisite for teachers to make a successful connection. In this study, teachers were mostly unable to show the desired achievement in the subcomponents requiring detailed anticipation, as responding to possible solutions or misconceptions and sequencing solution strategies concerning planning documents. Therefore, the teachers' lack of doing detailed planning prevented them from internalizing anticipated acting scenarios based on making connections between solution strategies.

5. Limitations and Future implications

In the current study, although worthwhile changes were observed in the classroom practices of two beginning teachers, some limitations affected the research. Accessibility of researchers to two far districts and sometimes the challenges of teachers' participation in meetings restrained the data collection process. We, indeed, anticipated such problems and completed the process by making additional meetings and observations. Besides, the students' low academic achievement, especially in Gizem's classroom, significantly affected the diversity and quality of the solution strategies of students. If the current study was carried out in a school with a higher academic success with a similar design, different results might emerge. On the other hand, the limitations of this study can also be considered as a significance of this study. Although academic success was low in such settings, we can transparently state based on the results of the study that when students have an opportunity to think and explore in mathematics lessons, significant changes may occur in the classroom athmosphere and perceptions, including teachers' beliefs and practices. The current study showed us that 5Ps could be used as a systematic tool both in undergraduate training and PDs for beginning teachers. This study also revealed the importance of making detailed planning based on student thinking and sustainable peer collaboration. For future studies, research focusing only on one or more components of 5Ps can be done, and the scope can be narrowed to a single learning domain.

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