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Research Article

Impact of multiple representations-based instruction on teaching and learning of linear equations

Kofi N. Mpuangnan[1], Blankson K. Adjei[2] and Samantha Govender[3]

[1]*Department of Curriculum & Instructional Studies, University of Zululand, South Africa;* [2]*University of Education Winneba, Ghana;* [3]*Department of Curriculum & Instructional Studies, University of Zululand, South Africa*

Correspondence should be addressed to Kofi N. Mpuangnan nkonkonya@gmail.com
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This paper investigates the impact of multiple representations-based instruction on the teaching and learning processes of linear equations among students in Standard VIII. It focuses on how different representations, such as graphs, tables, and equations, affect comprehension, retention, and overall mastery of linear equations in this educational context. An experimental design was employed, involving 159 students selected from Techiman Municipality in the Brong Ahafo Region of Ghana using a simple random sampling technique. Also, 86 mathematics teachers were randomly chosen to gather diverse viewpoints and valuable insights aimed at improving the teaching methods for this concept. Data collection instruments included a linear equations achievement test (pre-test & post-test) and a questionnaire. The collected data were analysed by using descriptive and inferential statistics. The study revealed that most teachers primarily relied on algebraic representation but only a few incorporated multiple representations due to various challenges such as time constraints, difficulty for students, lack of materials, and absence from the syllabus. It was further found that the implementation of multiple representations-based instructions resulted in a significant improvement in learners' scores in the linear equations achievement test, highlighting the effectiveness of this instructional approach. It was recommended that teachers teach linear equations in one variable using other representations such as manipulatives and graphics to enhance understanding. Also, students are encouraged to cultivate proficiency in integrating multiple representations when tackling problems related to linear equations. Further research should be conducted on equipping teachers with ample resources and designing robust training programs to enable them to adeptly incorporate multiple representations-based instruction for teaching linear equations.

Keywords: Ghana, linear equation, mathematics, multiple representation-based instructions, standard viii

1. Introduction

Individual learners possess varied learning styles, preferences, and strengths which require multiple representations to cater for their learning needs. Using multiple representations-based instructions [MRBI] for teaching particularly linear equations can provide unique insights into the concept. This teaching approach employs various methods of presenting the material, enabling students to comprehend the concepts of linear equations more easily (Doktoroglu, 2013). As linear equations are fundamental to developing learners' algebraic proficiency, it is essential to ensure that they grasp these concepts thoroughly (Huntley & Terrel, 2014). Despite the significance of linear equations, many students in Ghana still encounter challenges in developing a symbolic and conceptual understanding of the topic (Poon, & Leung, 2010). A study has shown that learners struggle specifically with equations in the form of $ax + b = cx + d$ (Gado & Adonteng-Kissi, 2016).

Standard VIII in the Ghanaian education system is the equivalent of 8th grade, and students in this level typically range from 13 to 14 years old (Mpuangnan & Adusei, 2021). Such students are either studying or completing six years of primary education. However, the available data from the West African Examination Council [WAEC] Chief Examiners' Report (2017) suggests that there is a concerning trend regarding the performance of Ghanaian learners in algebra, particularly in linear equations. The evidence points to the fact that most of the learners refrained from attempting questions involving linear equations. It has been reported that students struggle with manipulating algebraic variables (Mpuangnan et al., 2021). Additionally, the performance of such learners in mathematics, particularly in algebraic expressions and linear equations, highlights the difficulty they face in this subject (Anamuah-Mensah, & Mereku, 2005).

One possible explanation for this consistently poor performance could be attributed to the teaching methods employed by mathematics teachers in schools. It appears that learners may not be exposed to appropriate representations and techniques for understanding algebra and linear equations. As a result, they may not fully grasp the fundamental concepts, leading to difficulties during exams and in their overall academic performance in mathematics. To address this issue and improve teaching and learning outcomes in basic schools across Ghana, it has been suggested that a variety of teaching and learning materials should be implemented (World Bank, 2021; Mpuangnan, 2020). One promising approach to achieve this is by using a teaching technique based on multiple representations (Canterbury, 2007). It is anticipated that by incorporating various representations, such as graphical, algebraic, and tabular depictions, students can develop a deeper understanding of linear equations. This can further increase their confidence and proficiency in mathematics, and ultimately improve their performance in their final exams and beyond.

1.1. Multiple Representations

The concept of multiple representations involves the utilization of diverse mathematical formats or forms for instruction and problem-solving (NCTM, 2020). This basically refers to presenting the same information through multiple external mathematical forms. Consequently, the practice of employing various modes of representation by an educator to convey a particular concept can be described as the application of multiple representations. To illustrate, the teaching of linear equations in one variable can encompass the use of algebraic, manipulative, and graphic representations, all contributing to a comprehensive understanding.

A study conducted by DeJarnette et al. (2020) revealed that the utilization of multiple representations plays a pivotal role in enhancing knowledge reconstruction. These representations serve as essential tools for fostering conceptual learning in the fields of mathematics and science. Larson et al. (2022) highlighted the significance of a diversified approach in comprehending fractions (such as $1/3$), asserting that students should possess the ability to discern this concept through various forms of representation. Similarly, the grasp of linear equations, exemplified by equations such as $x + 2 = 2x - 7$, $x + 4 = 10 + 4x$, $2(x + 3) = -3x - 4$, $\frac{1}{2}x + 5 = 2x + 3$, $\frac{1}{4}x + 4 = -\frac{1}{2}x - 1$, $\frac{1}{3}x - 3 = -\frac{5}{3}x + 3$, and others, can be attained more effectively when students are exposed to a diverse array of representations.

1.2. Representational Modes for Teaching Linear Equations

In this study, the researchers aim to explore various representational modes for teaching "two-step equations," which are linear equations that involve a variable (pronumeral) on both sides of the equation. The representational modes under review are algebraic, manipulatives, and graphic and tabular representations. These modes were used to facilitate the understanding and learning of equations in the form of $ax \pm b = cx \pm d$, where a, b, c, and d are constants, and x is the pronumeral (variable). Through these different modes, effective teaching strategies that can enhance students' comprehension and proficiency in solving such equations would be sought.

1.2.1. Algebraic representation

Rethinking the approach, algebraic representation has stood the test of time as a classic method for teaching linear equations in one variable. Rooted in tradition, it relies on a systematic set of rules and procedures. These procedures encompass various operations such as addition, subtraction, combining similar terms, distribution, factoring, and multiplication or division by variables or constants (Star, 2005). Notably, educators and teachers have displayed a clear preference for algebraic representation when imparting mathematical concepts (Larson, et al., 2022). Within this framework, both teachers and students employ a range of strategies to tackle problems involving linear equations. These strategies encompass utilizing number facts, employing counting techniques, employing the cover-up method, undoing operations, transposing elements, and achieving balance (Kieran, 1992).

In some instances, solving linear equations entails determining all feasible values that can replace a variable within an equation. To undertake this task, the process involves substituting numerical values into the algebraic expression on both sides of the equation and assessing the results. For example, when confronted with the equation 5x+3=2x+15 and seeking to solve for x, the initial step involves substituting x=0, resulting in [5(0) +3=3, 2(0) +15=15]. It quickly becomes evident that this statement is incorrect, as the Left-Hand Side does not align with the Right-Hand Side. This leads to a systematic exploration of other potential values like 1, 2, 3, and 4 until the realization emerges that the statement becomes valid when x=4. However, a study conducted by (Matz, 1981) emphasized the limitation of seeking solutions by iteratively testing different values of x. In this process, the focus tends to shift away from manipulating algebraic expressions and instead centres on manipulating their numerical representations. To progress beyond this method, a shift towards a more structural approach becomes imperative. This novel perspective involves executing distinct sets of operations, not on mere numerical quantities, but rather on the underlying algebraic expressions themselves as in:

$$2x - 6 = x + 8$$
$$2x - 6 + 6 = x + 8 + 6$$
$$2x = x + 14$$
$$2x - x = x - x + 14$$
$$x = 14$$

It could be noted that the structural approach helps students grasp the structure of an equation and become successful in solving equations. Similarly, (Bittinger et al., 2013) identified two kinds of processes involved in solving first-degree equations deduction and reduction. The deduction involves performing the same operation on both sides of the equal sign while reduction deals with replacing one expression with another equivalent expression as in:

$$3x + 7 = 2x\ ,$$
$$3x + 7 - 2x = 2x - 2x\ (deduction),$$
$$x + 7 = 0\ (reduction),$$
$$x + 7 - 7 = 0 - 7(deduction),$$
$$x = -7(reduction).$$

Rethinking the process of solving linear equations reveals the balancing method as a central technique. This method hinges on maintaining equilibrium by performing identical operations on both sides of the equation. Regrettably, students frequently struggle to grasp this concept, necessitating dedicated instruction. Moreover, research highlights that the introduction of negative numbers complicates matters when employing the balancing method (Leung et al., 2014).

Furthermore, investigations into teaching practices have exposed shortcomings in conveying essential mathematical concepts. For instance, during a classroom observation detailed in (Sidney, 1993), a teacher emphasized the directive "do the same to both sides" while unravelling the equation $3x + 2 = 2x + 3$. This approach, while providing a rule of thumb, failed to illuminate the mathematical rationale behind subtracting 2x from both sides ($3x - 2x + 2 = 2x - 2x + 3$). Notably absent was an explanation of the underlying inverse properties of operations, which are pivotal in undoing operations. To elaborate, addition (+) serves as the reverse of subtraction (-), and vice versa. Similarly, multiplication (×) acts as the counterpart to division (÷), and the two operations can nullify each other (Lima & Tall, 2008). For instance, subtractions undo addition as in:

$$x + 4 = 6$$
$$x + 4 - (4) = 6 - (4)$$
$$x = 2$$

Also, additions undo subtraction as in:

$$x - 2 = 5$$
$$x - 2 + (2) = 5 + (2)$$
$$x = 7$$

Again, divisions undo multiplication as in:

$$3x = 9$$
$$\frac{3x}{3} = \frac{9}{3}$$
$$x = 3$$

Lastly, multiplications undo division as in:

$$\frac{x}{4} = 5$$
$$4 \times \frac{x}{4} = 4 \times 5$$
$$x = 20.$$

A study conducted by (Cortes & Pfaff, 2000) focused on students' perspectives on equations, examining both their conceptual understanding and solution strategies. The findings suggested that the process of solving equations was often perceived as a mere manipulation of symbols. Moreover, Cortes and Pfaff (2000) observed that the 17-year-old participants in their research predominantly employed a symbol-shifting approach to equation solving. This approach seemed to treat mathematical symbols as tangible entities, capable of being relocated across the equal sign along with sign alterations, akin to physical objects in motion. For instance, the equation $2x - 6 = -x - 12$ can be solved by transposing the terms as in:

$$2x - 6 = -x - 12$$
$$2x + x = -12 + 6$$
$$3x = -6$$
$$\frac{3x}{3} = \frac{-6}{3}$$
$$x = -2.$$

On the other hand, Li (2007) demonstrated that employing a multiple-representation approach yields enhanced efficiency and speed, although it may sacrifice transparency. Consider the process of solving equations like $2x + 3 = x + 8$; it involves a sequence of seemingly arbitrary steps, such as transferring the 3 across the equal sign and altering its sign to obtain $2x = x + 8 - 3$. These steps can appear devoid of meaningful understanding. This prompts us to ponder: "What is the rationale behind moving the 3 across the equal sign?" and "Why should this action entail changing the sign of the 3?" In this context, crucial questions arise. Rather than performing a superficial manipulation, why not foster the notion that mathematics revolves around sensible actions? Encouraging students to contemplate, "I can subtract 3 from both sides of the equation without altering its solution set," would likely yield more meaningful insights, as Freitas (2002, as cited in de Lima and Tall, 2008) underscored in their investigation. The utilization of phrases like "change side" or "change sign" often proves bewildering to students, leading to errors. Nevertheless, there is another representation called graphical representations. Both algebraic and graphical representations have their strengths and applications. Algebraic representations are excellent for deriving precise and generalized solutions, while graphical representations excel at providing visual insights and aiding in problem-solving (Huntley et al., 2007; Ronda & Dzoba, 2022). These two methods complement each other and are interconnected tools for exploring and understanding mathematics.

2. Research Questions

The researchers seek to find answers to the following questions:

RQ 1) To what extents do standard viii teachers use multiple representations-based teaching approaches in teaching linear equations?

RQ 2) What factors influence the use of multiple representations-based teaching approaches in standard viii?

RQ 3) To what extent can teachers use multiple representations-based teaching to make a significant difference between students' scores in linear equations' achievement tests and traditional instruction after controlling for students' age, gender and pre-test scores?

3. Method

3.1. Research Design

The study was carried out through experimental design, involving the collection of quantitative data from the participants. This method involves selecting two similar groups of Standard VIII students, one exposed to traditional teaching methods and the other to MRBI. Before the intervention, each group would take pre-tests to assess their understanding of linear equations. The MRBI group would then be instructed using various representations such as graphs, tables, and equations, while the control group would follow conventional teaching approaches. Subsequent post-tests would evaluate their understanding and retention of linear equations. For a reliable comparison of MRBI's effectiveness, the study carefully managed several variables. Prior knowledge levels in both groups were balanced before the intervention, ensuring a fair assessment of the teaching method's impact without being influenced by initial disparities in understanding. Teachers with a bachelor of education in mathematics were selected to standardize teaching experience across the study groups. Additionally, the classroom environment was kept consistent, including factors like class size, available resources, and teacher-student interactions. These measures aimed to create a level playing field, allowing the evaluation to focus more accurately on the effects of MRBI itself rather than other external factors.

3.2. Participants

In this study, the focus was on Junior High School [JHS] learners, with a specific emphasis on those at the JHS 2 (Standard VIII) level. The JHS curriculum aims to foster an inclusive and equitable educational experience for all learners. To ensure a comprehensive representation, a systematic sampling approach was utilized to select three schools: School A, School B, and School C. Additionally, a random sampling technique was applied to choose a total of 159 learners from these schools. Among the selected schools, the demographic distribution was as follows: School A had 24 boys and 29 girls, School B had 16 boys and 37 girls, and School C had 28 boys and 25 girls. The intention was to ensure that the chosen learners possessed comparable levels of training and achievement in mathematics, making them suitable participants for this study. Moreover, the schools selected boasted professional mathematics teachers employed by the Ghana Education Service, negating the need for supplementary training during the study period. To further gain insights into teaching methods and strategies employed in mathematics classrooms, the researchers randomly selected 86 mathematics teachers from the Techiman Municipality, anticipating that they could provide valuable data on teaching methodologies. While the teachers themselves were not actively involved in the experiment, their perspectives on teaching the concept were gathered to seek insights for potential improvements. This research carried paramount significance at this particular stage, considering that the learners were expected to have covered mathematics content up to the level of algebraic equations, including $ax \pm b = c$, $ax \pm b = cx$, and $ax \pm b = cx \pm d$.

The researchers utilized a systematic sampling method to categorize schools into specific groups for the study. This approach involved assigning a distinct numerical identifier (Creswell, 1994), to each school in facilitating the selection of a subset of schools to be included in the study.

As a result, School A was designated as Experimental Group 1, School B as Experimental Group 2, and School C as the Control Group. To achieve the aim of the study, the researchers designed three distinct stages of instruction to teach the learners. The first stage involved School A (Experimental Group 1) and encompassed instruction with three representations: algebraic, manipulative, and graphic. The second stage, for Experimental Group 2 (School B), focused on two representations: algebraic and manipulatives. In contrast, the third stage comprised the Control Group (School C) receiving traditional instruction relying solely on algebraic representation, without integrating additional visual or manipulative aids. The researchers employed a linear equations achievement test [LEAT] in both a pre-test and a post-test stages, to evaluate the effectiveness of the instructions, and measure the learners' proficiency in problem-solving. LEAT was used because Algebraic Equations can be linear or nonlinear (Linge & Langtangen, 2016). To provide ample time for implementing the intervention, the experiment extends across two semesters, totalling a period of six to eight months. More information about the assessment process will be provided in the subsequent section. The purpose the the instructions was to compare the effects of having fewer representations than in Experimental Group 1. Smith and Thompson (2020) argue that traditional approaches may impede students' conceptual understanding and hinder their grasp of abstract mathematical concepts, especially for those who learn better through visual or hands-on experiences. The details of the students' scores are illustrated in Figure 1 and Figure 2, respectively.

Data about the teachers’ background covered characteristics such as; gender status, age, professional status, qualification and several years served as a mathematics teacher. All these were done to solicit in-depth information from teachers who were involved in the study. Data gathered on teachers' demographic characteristics are presented in Table 2.

Table 2

Summary of Demographic Characteristics of Teachers

Demographic factors	*Category*	*Frequency*	*Percentage (%)*
Gender	Male	68	79.1
	Female	18	20.9
	Total	86	100.0
Age	20-25 years	7	8.1
	26-30 years	36	41.9
	31-35 years	28	32.6
	36-40 years	11	12.8
	41 years and above	4	4.7
	Total	86	100.0
Professional Status	Pupil-teacher	1	1.2
	Non-professional	8	9.3
	Professional	77	89.5
	Total	86	100.0
Academic Qualification	SSCE/WASSCE	1	1.2
	Certificate ‘A’	2	2.3
	Diploma	33	38.4
	HND	6	7.0
	Degree (B.Sc./Bed etc.)	38	44.2
	Masters	6	7.0
	Total	86	100.0
Years of Teaching Mathematics	1-5 years	45	52.3
	6-10 years	26	30.2
	11-15 years	10	11.6
	16-20 years	3	3.5
	21 years and above	2	2.3
	Total	86	100.0

Table 2 presents an insightful breakdown of the gender distribution, age range, professional status, academic qualifications, and years of experience among mathematics teachers in the Techiman Municipality. The data offers valuable insights into the composition of the teaching workforce in the context of mathematics education. Gender-wise, the data illustrates a noticeable gender disparity among mathematics teachers. Specifically, 68 (79.1%) of the participating teachers were male, whereas 18 (20.9%) were female. This indicates a higher representation of male teachers in mathematics at the junior high school [JHS] level within the Techiman Municipality during the study period. Shifting our focus to teachers' age, the statistics reveal that the majority of teachers fell within the youthful age brackets of 26 to 30 years and 31 to 35 years, constituting 36 (41.9%) and 28 (32.6%) teachers, respectively. In contrast, the number of teachers aged 20 to 25 years and 41 years and above was relatively small, accounting for only 7 (8.1%) and 4 (4.7%) teachers, respectively. This distribution highlights the concentration of teachers within certain age ranges.

Regarding professional status, a substantial proportion of the teachers, specifically 77 (89.5%), were classified as professional teachers, underscoring the prevalence of trained educators within the sample. Interestingly, there was only one pupil-teacher, representing a mere 1.2% of the total. This indicates that most teachers participating in the study possessed formal teaching qualifications. Turning our attention to academic qualifications, the data portrays that 33 (38.4%) teachers held Diplomas, while 38 (44.2%) held Bachelor's degrees, collectively constituting the dominant academic attainments among the teachers. In contrast, only 1 (1.2%) teacher possessed an SSSCE/WASSCE qualification and 6 (7.0%) teachers boasted master's degrees and HND certificates. This distribution showcases the prevalence of Diploma and Bachelor's degree holders among the mathematics teaching cohort. Exploring the number of years taught by mathematics teachers, it becomes evident that 45 (52.3%) teachers had taught mathematics for a duration ranging from 1 to 5 years, while 26 (30.2%) teachers had accumulated 6 to 10 years of teaching experience. This suggests that a significant portion of teachers had taught mathematics for relatively shorter periods within these ranges. In contrast, a few 2 (2.3%) teachers possessed extensive experience, having taught mathematics for 21 years or more.

3.3. Instruments

The study employed a combination of data collection methods, including a questionnaire featuring close-ended questions. Also, a linear equations achievement test (LEAT) was used. The questionnaire was thoughtfully designed for teachers to ensure comprehensive data gathering while minimizing costs and effort (Osula, 2001). Therefore, the questionnaire was comprised of two distinct sections. The initial section (Section A) focused on gathering demographic information about the participating educators. This encompassed a diverse range of attributes, such as gender, age, professional status, qualifications, and years of experience as a mathematics teacher. The subsequent section (Section B) of the questionnaire delved into the teaching practices of educators regarding linear equations. Four questions (Q6-Q9) were presented to the participants. Specifically, Question 7 featured five sub-questions, each about a specific mode of representation employed in teaching linear equations. These sub-questions were rated on a Likert scale, allowing teachers to indicate the frequency with which they used each mode of representation, ranging from "1=never" to "5=every time." This facilitated an exploration of the various approaches teachers employed when teaching linear equations. Additionally, Questions 8 and 9 contained five sub-questions each, aimed at investigating the factors influencing educators' decisions regarding their choice of representations when teaching linear equations.

The assessment of linear equation achievement involved both a pre-test and a post-test, carefully designed by the researchers to determine students' proficiency in solving linear equations with a single variable. These evaluations were carried out across three groups, both before and after teaching instruction using multiple representations. To initiate the process, students from all three groups participated in a collaborative exercise during the pre-test. This collaborative activity encouraged active engagement in solving linear equations, fostering a deeper understanding of the topic (Kaur & Drijvers, 2021). Subsequently, the researchers employed a series of ten multiple-choice questions to assess the student's comprehension of linear equations. The questions were designed to effectively measure their grasp of the subject matter. Students selected their answers from options labelled A to D. For the post-test, all three groups were examined again after receiving instructions centred around multiple representations. The format and number of questions mirrored those of the pre-test, but the specific questions differed. The values and figures used in the questions were modified, ensuring that the post-test accurately measured the student's ability to apply their knowledge rather than simply recalling specific information.

3.4. Pilot Study

In this study, the researchers piloted all the research instruments in the Techiman North District in the Brong East Region of Ghana. The questionnaire was piloted with 30 classroom mathematics teachers. Also, the linear equations’ achievement test (pre-test and post-test) and the linear equations’ achievement test were piloted with 30 junior high students in the Akrofrom R/C JHS. It took students 17 minutes instead of 20 minutes of the given time to complete the linear equations’ achievement test.

3.5. Validity and Reliability

The process of ensuring the validity of the instrument involved both content and faces validity assessments. For the linear equations' achievement test's content validity, various authoritative sources were utilized, including the JHS 2 mathematics course book, the teaching syllabus, and pertinent dissertation works. These references provided a comprehensive foundation for the test's content, aligning it with the intended student learning outcomes. To enhance the content validity further, the developed test was scrutinized by three experts in mathematics education. These experts possessed substantial expertise in the field, with one holding a Doctorate in mathematics education and more than ten years of professional experience, while the other two were distinguished professors in mathematics education with over two decades of academic and practical involvement. It is noteworthy that uniformity was maintained across the curriculum, syllabus, and textbooks employed by educators in the study's chosen schools. This meticulous selection was made to eliminate potential biases and discrepancies in the gathered data, thus fortifying the study's credibility and ensuring the reliability of the instrument.

After conducting a pilot study on the instruments, the internal consistency of the questionnaire and the achievement tests' linear equations were assessed for reliability. This was achieved by calculating the reliability coefficient using both Cronbach's alpha and the Split-half method. The software SPSS was utilized to compute the reliability coefficients for all the instruments. In the Split-half method, the scores of students in the pre-test were divided into two separate halves and then scored accordingly. The same process was repeated for the post-test. For each participant, scores were determined for odd-numbered items as well as even-numbered items. The estimation of reliability coefficients was carried out using SPSS. The Spearman-Brown coefficient yielded reliability coefficient values of 0.71 for the pre-test and 0.72 for the post-test. Additionally, the questionnaire displayed a Cronbach's alpha (α) of 0.76, indicating its reliability. These calculated reliability estimates, falling within these ranges, were deemed trustworthy (George & Mallery, 2003). Consequently, it is appropriate to consider the instruments used in this study as reliable.

3.6. Data Analysis

The data collected underwent analysis through a combination of descriptive and inferential statistical methods, facilitated by the utilization of SPSS Version 20 and Microsoft Excel 2013. In examining the data for research question one, descriptive statistics including measures such as mean, standard deviation, frequencies, and percentages were employed. For research question two, the analysis focused on frequencies and percentages exclusively. The investigation of research question three encompassed a broader range of techniques, encompassing frequencies, percentages, graphical representations, Pearson Product-Moment Correlation, and One-Way ANCOVA (Analysis of Covariance).

4. Results

4.1. Grade Eight Teachers' Mode of Using Multiple Representations-based Teaching Approaches in Teaching Linear Equations

Table 3 presents the data showing the extent to which teachers use multiple representations-based teaching approaches in teaching linear equations. The scale of responses in Table 3 was condensed to the following categories: Never (N), Almost never (AN), Occasionally (O), Almost every time (AE), and Every time (ET).

Table 3

Teachers' mode of representation on linear equations in one variable

Description of mode of	*Frequency/Percentage*						
Representations	N	AN	O	AE	ET		
	N (%)	N (%)	N (%)	N (%)	N (%)	Mean	SD
Algebraic	0(0%)	0(0%)	0(0%)	29(33.7%)	57(66.3%)	4.66	0.48
Manipulatives	32(37.2%)	29(33.7%)	4(4.7%)	15(17.4%)	6(7.0%)	2.23	1.31
Graphic	43(50.0%)	19(22.1%)	14(16.3%)	8(9.3%)	2(2.3%)	1.92	1.12
Multiple representations	19(22.1%)	13(15.1%)	26(30.2%)	16(18.6%)	12(14.0%)	2.87	1.34
Single representation	0(0%)	2(2.3%)	11(12.8%)	34(39.5%)	39(45.3%)	4.28	0.78

The findings presented in Table 3 reveal that a significant portion of the sampled teachers, 57 (66.3%), consistently employed algebraic representation when teaching. According to the data, almost all the teachers applied algebraic or single representations for linear equations. A smaller group of teachers, comprising 6 (7.0%) teachers, employed manipulatives regularly, while graphics were used by 2 (2.3%) teachers, and multiple representations were employed by 12 (14.0%) teachers.

4.2. Factors Influencing the Use of Multiple Representations-based Instructions for Teaching Linear Equation

Teachers' choice of instruction method might depend on certain reasons or factors. To study such factors in this study, data were collected from two different perspectives. The first perspective was teachers' reasons for using multiple representations-based instructions and the second perspective was teachers' reasons for not using it. The details of the findings are presented as follows.

4.2.1. Teachers' reasons for using multiple representations-based instructions

To examine why teachers would prefer to use multiple representations-based instructions for teaching linear equations, data were collected from mathematics summarised in Table 4. Table 4 depicts a revealing insight into teachers' perspectives on algebraic representation. Among the surveyed teachers, a significant majority of 54 (62.7%) find algebraic representation to be not only straightforward but also comprehensible, demonstrating its accessibility. Furthermore, 12 (14.0%) teachers noted that it holds the distinction of being a widely recognized and frequently employed method. In the realm of teaching linear equations, those who incorporate manipulatives and graphic representations into their pedagogical approach provided noteworthy rationale. A notable 13 (52.0%) teachers endorsed manipulatives, emphasizing their ability to enhance comprehension. Similarly, 10 (41.6%) educators favoured graphic representations for their capacity to facilitate understanding. Among those who embraced the use of multiple representations, 17 (31.5%) educators highlighted its effectiveness in fostering understanding, underscoring its educational value. An additional 14 (25.9%) teachers acknowledged that multiple representations serve as a source of motivation for their students. Conversely, for those who opted for a single representation approach, their reasons closely paralleled those expressed in favour of algebraic representation.

Table 4
Reasons for teachers' mode of representation in linear equations

Representations	*Reasons*	*Frequency*	*Percentage (%)*
Algebraic	Easy, Simple and Understandable	54	62.7
	Pupils have knowledge of it	8	9.3
	Well-known representation	12	14.0
	Faster	4	4.7
	Others	8	9.3
	Total	86	100.0
Manipulatives	Better understanding	13	52.0
	Makes lessons real	10	40.0
	Others	2	8.0
	Total	25	100.0
Graphic	Better understanding	10	41.6
	Makes lesson practical	7	29.2
	Others	7	29.2
	Total	24	100.0
Multiple representations	Promotes understanding	17	31.5
	Motivates students	14	25.9
	Address different learning styles	12	22.2
	Others	11	20.4
	Total	54	100.0

4.2.2. Teachers' reasons for not using certain representations in teaching linear equation

Data on why teachers do not use certain representations in teaching linear equations were also collected and presented in Table 5.

Table 5
Reasons for teachers not using certain modes of representations in linear

Representations	*Reasons*	*Frequency*	*Percentage (%)*
Manipulatives	Time-consuming	9	14.8
	Difficult to understand	10	16.4
	Lack of materials	19	31.1
	Have no idea	16	26.2
	Others	7	11.5
	Total	61	100.0
Graphic	Time-consuming	13	21.0
	Difficult to understand	24	38.6
	Not found in the syllabus	13	21.0
	Have no idea	8	12.9
	Others	4	6.5
	Total	62	100.0
Multiple representations	Time-consuming	13	40.5
	Lack of materials	8	25.0
	Students get confused	3	9.4
	Have no idea	6	18.8
	Others	2	6.3
	Total	32	100.0

The results in Table 5 reveal that among the surveyed group of 61 teachers who do not integrate manipulatives into their teaching methods for linear equations, 31.1% (19 teachers) attribute this choice to a scarcity of available materials. Additionally, 26.2% (16 teachers) expressed a lack of familiarity with the effective utilization of manipulatives in teaching linear equations. A cumulative 11.5% cited alternative factors such as financial constraints, non-alignment with the syllabus, and perceived irrelevance to student needs. Within the cohort of 62 teachers who abstain from using graphic representation, 38.6% (24 teachers) highlighted student difficulties in comprehending this approach as a primary concern. Moreover, 21.0% (13 teachers) identified the time-intensive nature of graphic representation, coupled with its absence from the prescribed syllabus, as influential factors. A combined 6.5% cited reasons such as financial implications, material unavailability, and limited representation in examination questions. Exploring the realm of multiple representations, among the 32 teachers who refrain from their use, 40.5% (13 teachers) point to the time demands associated with this technique. Furthermore, 25.0% (8 teachers) attributed their avoidance of multiple representations to a scarcity of necessary materials. However, in the case of algebraic representation, an overwhelming majority of teachers embraced this approach, rendering the provision of reasons unnecessary. This trend paralleled that observed with single representation methods.

4.3. Difference between Students' Scores in Linear Equation Achievement Test and Traditional Instruction

To determine the difference between students' scores in linear equations achievement tests and traditional instruction, the descriptive statistics of students' pre-test and post-test scores were found and presented in Table 6.

Table 6
Mean Scores of students' pre-test and post-test by group

Variable	*Group*	*Mean*	*SD*	*Min*	*Max*
Pre-test	Experimental group 1	3.13	1.13	1	6
	Experimental group 2	3.02	1.84	0	9
	Control group	2.91	1.04	1	6
Post-test	Experimental group 1	4.11	1.71	1	9
	Experimental group 2	3.23	2.30	0	9
	Control group	2.51	1.42	0	6

Table 6 sheds light on the shifts in student performance across different groups. Specifically, the pre-test mean scores of students in Experimental Group 1 displayed an upward trajectory (initial mean M=3.13, SD=1.13), culminating in higher post-test mean scores (final mean M=4.11, SD=1.71). A similar trend was observed in Experimental Group 2, where the pre-test mean scores (M=3.02, SD=1.84) were surpassed by post-test mean scores (M=3.23, SD=2.30). Conversely, the Control group's pre-test mean scores (M=2.91, SD=1.04) failed to exhibit a corresponding increase in post-test mean scores (M=2.51, SD=1.42). It is noteworthy that both Experimental Group 1 and the Control group exhibited identical minimum and maximum pre-test values (1 & 6). Additionally, the maximum post-test values for Experimental Group 1 and Experimental Group 2 were 9. These data indicate that, at the outset, the students in Experimental Group 1 and the Control group achieved similar scores; however, post-intervention, the Experimental groups demonstrated notable improvements, attaining the highest scores.

Supplementing the descriptive statistics, we constructed boxplots for students' pre-test and post-test scores, visually represented in Figures 1 and 2. Analysing the pre-test scores (Figure 1) uncovers an equivalent median value for Experimental Group 1 and the Control group. In contrast, the post-test scores boxplot (Figure 2) underscores Experimental Group 1's pre-eminence in median value within the groups. This observation underscores the substantial enhancement in student achievement within Experimental Group 1, resulting from the intervention's implementation.

A Simple box plot showing Students' Pre-test Scores and post-tests is presented in Figure 1 and Figure 2 below.

Figure 1

A Simple Boxplot for Students' Pre-test Scores

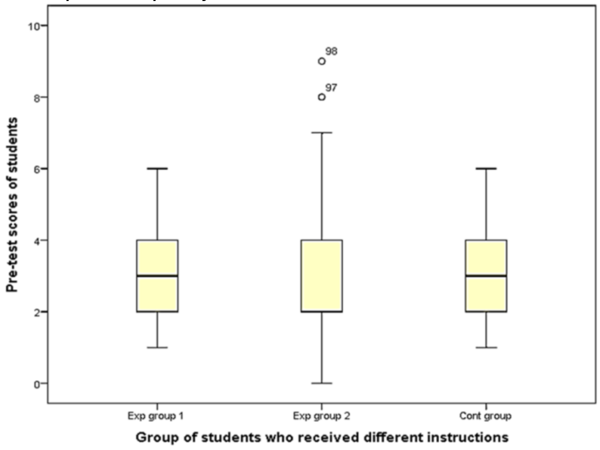
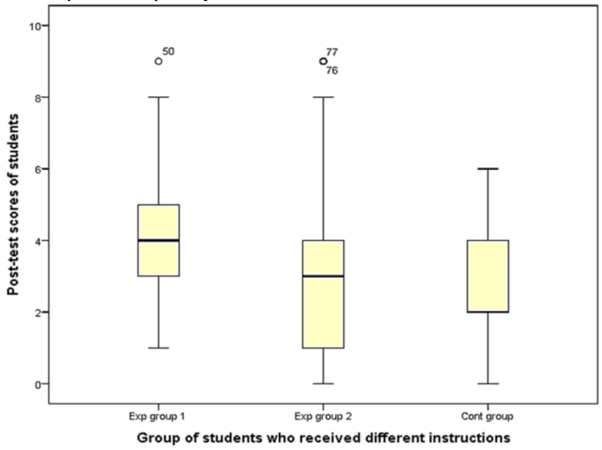


Figure 2

A Simple Boxplot for Students' Post-test Scores



To test the hypothesis, inferential statistics via one-way ANCOVA (Analysis of Covariance) was used, to check the following assumptions: 1) Independence of observation, 2) normality, 3) measurement of the covariates, 4) reliability of the covariates, 5) correlation between the covariates and the dependent variable, 6) linearity, 7) homogeneity of variance, and 8) homogeneity of regression (slopes).

The researchers personally conducted and oversaw both the pre-test and post-test phases, ensuring that each student responded independently, thus ensuring the observation of independence. To assess the assumption of normality, histograms with normality curves were employed for the pre-test and post-test scores, depicted in Figure 3 and Figure 4 respectively. The histograms revealed a central clustering of bars, indicative of a reasonably normal distribution of scores. Furthermore, the measurement of covariates took place before the introduction of the intervention. The pre-test exhibited a reliability coefficient of 0.71. The outcomes of the correlation analysis between the covariates and the dependent variable, i.e., students' post-test scores, are detailed and presented in Table 7.

Table 7

Correlations between the covariates and the dependent variable

	Correlations	
		Students' post-test scores
Students' pre-test scores	Pearson Correlation	0.512**
	Sig. (2-tailed)	.000
	N	159
Students' gender	Pearson Correlation	−.131
	Sig. (2-tailed)	.100
	N	159
Students' age in years	Pearson Correlation	0.170*
	Sig. (2-tailed)	.032
	N	159

The analysis presented in Table 7 demonstrates the associations between various factors and the dependent variable. Specifically, a positive correlation exists between students' pre-test and post-test scores, with a moderate correlation coefficient of r =0.51 ($p < .01$), aligning with the characterization of this correlation as medium or reasonable (Larbi, & Okyere, 2014). Regarding the relationship between students' age and post-test scores, a small correlation is observed ($r = 0.17$, $p < .05$). However, this correlation does not reach a level of significance for students' gender and post-test scores ($r = -0.13$, $p = .10$). Consequently, both students' age and gender are excluded as covariates from the analysis.

To ensure the assumptions of the analysis are met, we assessed the linearity among the variables. The scatterplot of students' pre-test and post-test scores (Figure 3) indicates a linear relationship without violating the assumption of linearity. This observation is supported by the straight lines in the plot. Similarly, the scatterplot of students' age and post-test scores (Figure 4) demonstrates no violation of the linearity assumption, despite a weak relationship between these variables. Conversely, the scatterplot of students' gender and post-test scores (Figure 5) depicts a non-linear relationship due to the intersecting lines, which indicates a violation of the assumption of linearity. Based on this, we omit students' age and gender as covariates from the analysis. Consequently, only students' pre-test scores are considered as a covariate in the current study.

Figure 3

Test of Linearity between Students' Pre-test and Post-test Scores

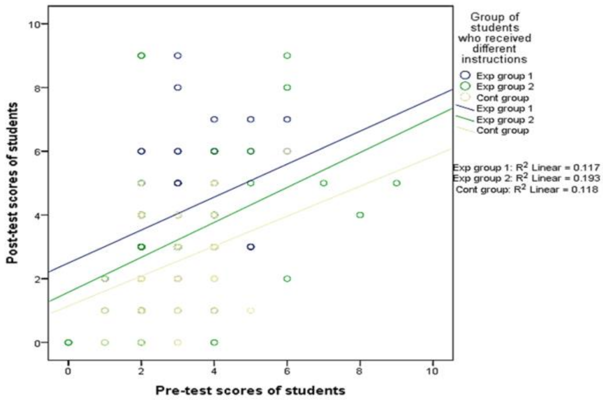


Figure 2

Test of Linearity between Students' Age and Post-test Scores

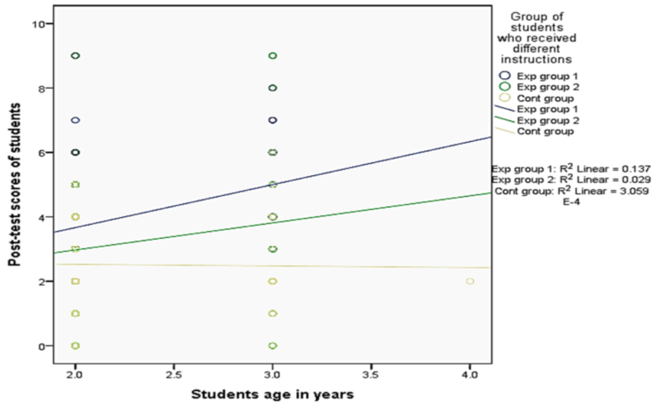
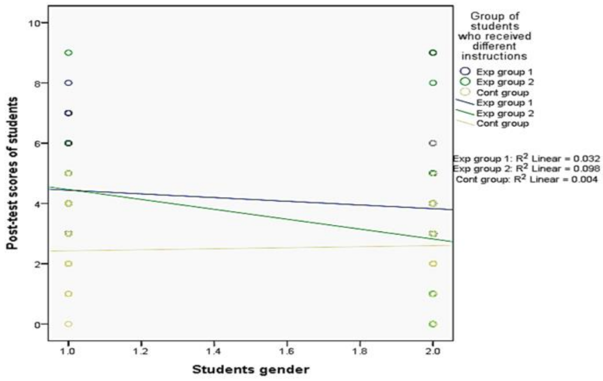


Figure 5

Test of Linearity between Students' Gender and Post-test Scores



It was further required to examine the assumption of homogeneity of variance to ensure that the variance of post-test scores across the groups was equal as presented in Table 8.

Table 8

Levene's Test of Equality of Error Variance (Dependent variable: Post-test scores of students)

F	*df1*	*df2*	*Sig.*
2.784	2	156	.065

Note. a. Design: Intercept + Pre-test + Group

The findings presented in Table 8 demonstrate that the assumption of homogeneity of variance is upheld, as indicated by the $F(2, 156) = 2.784$ with a p-value greater than .05. This indicates that the variability in post-test scores remains consistent across all groups, affirming the fulfilment of this assumption. Additionally, an assessment was conducted to verify the assumption of homogeneity of regression. This step aimed to confirm the absence of any interaction between the pre-test scores and the groups, ensuring the validity of the analysis. The results are presented in Table 9.

Table 9
Interaction between the pre-test scores and the group (Dependent Variable: Post-test scores of students)

Source	*Type III Sum of Squares*	*df*	*Mean Square*	*F*	*Sig*
Group	5.909	2	2.954	1.011	.366
Pretest	61.731	1	61.731	21.120	.000
Group* Pretest	0.267	2	0.134	0.046	.955
Error	447.202	153	2.932		
Corrected Total	598.692	158			

Note. a. R Squared = 0.253 (Adjusted R Squared =0.229).

The findings presented in Table 9 demonstrate that the assumption of homogeneity of regression remains unchallenged, as indicated by the statistical values $F(2, 153) = 0.046$, $p > .05$. This implies that no discernible interaction exists between the pre-test scores and the respective groups, thereby confirming the fulfilment of the assumption. With the preliminary conditions met, the researcher proceeded to conduct a One-way Analysis of Covariance (ANCOVA) to examine the research hypothesis. The independent variable, encompassing three distinct levels—Experimental Group 1, Experimental Group 2, and Control Group—was analysed. In this investigation, the students' pre-test scores were utilized as a covariate, while their post-test scores served as the dependent variable. The ANCOVA results are reported in Table 10.

Table 10
Results of ANCOVA for post-test scores of students (Dependent Variable: Post-test scores of students)

Source	*Type III Sum of Squares*	*df*	*Mean Square*	*F*	*Sig*	*Partial Eta Square*
Pretest	82.339	1	82.339	28.522	.000	0.155
Group	58.897	2	29.449	10.201	.000	0.116
Error	447.470	155	2.887			
Corrected Total	598.692	158				

Note. a. R Squared = 0.253 (Adjusted R Squared =0.238).

Upon ensuring that the assumption of homogeneity of regression was met to eliminate any potential interaction between pre-test scores and groups, the analysis in Table 10 revealed the following outcomes: Firstly, the relationship between pre-test scores and post-test scores was examined, with the analysis yielding a statistically significant relationship (F (1, 155) = 28.52, $p < .05$, $\eta^2 = 0.15$). Additionally, while considering the remaining assumptions outlined in Table 10, an ANCOVA was conducted. This ANCOVA indicated a noteworthy divergence among groups in terms of students' achievement scores ($F(2, 155) = 10.20$, $p < .05$, $\eta^2 = 0.12$). This effect size of 12% in post-test score variance aligns with a substantial impact, as defined in (Cohen et al., 2008). To ascertain the group differences, post hoc tests were employed, specifically the LSD pairwise comparisons among the adjusted means. Detailed outcomes of these pairwise comparisons are provided in Table 11.

Table 11
ANCOVA pairwise comparisons of the adjusted means among the groups (Dependent Variable: Post-test scores of students)

Groups	*Adjusted Mean*	*Comparisons*	*Mean Difference*	*Std. Error*	*Sig*
Exp group 1	4.05	Exp group 1 Vs. Exp group 2	0.82*	0.330	.014
Exp group 2	3.23	Exp group 1 Vs. Cont group	1.50*	0.329	.000
Cont group	2.60	Exp group 2 Vs. Cont group	0.66*	0.332	.048

Note. *. The mean difference is significant at the .05 level.

The outcomes presented in Table 11 highlight notable distinctions among the student groups. Notably, participants in Experimental Group 1 exhibited the highest adjusted mean score (M = 4.05), followed by Experimental Group 2 (M = 3.23), while the control group displayed the lowest mean (M = 2.60). Additionally, a thorough analysis of the adjusted mean scores through LSD pairwise comparisons demonstrated significant statistical disparities in students' achievement scores between Experimental Group 1 and Experimental Group 2 (M = 0.82, $p = .014$), Experimental Group 1 and Control group (M = 1.50, $p = .000$), as well as Experimental group 2 and Control group (M = 0.66, $p = .048$). Notably, these differences remained significant even after accounting for the influence of students' pre-test scores. The ANCOVA outcomes further reinforce these findings, revealing a statistically significant disparity in students' scores on the linear equations' achievement test between the groups using multiple representations-based instructions and those using traditional instruction ($F(2, 155) = 10.20$, $p < .05$, $\eta^2 = 0.12$). This underlines the impact of the innovative multiple representations-based instruction method on enhancing students' achievement.

5. Discussion

The results of the study regarding the utilization of various teaching approaches for linear equations indicate a prevailing preference for algebraic representation among teachers, with a substantial majority (66.3%) consistently employing this method. The study also highlights a significant trend among sampled teachers, who predominantly employ multiple representation techniques when instructing on linear equations. These findings resonate with prior research (Hitt, & Trinterud, 2019) and (Bal, 2014), underscoring the prevalent use of algebraic representation in mathematical problem-solving pedagogy (Cai, 2005). The rationale behind this inclination towards algebraic representation appears rooted in its perceived clarity and efficiency, with educators finding it comprehensible and quicker to implement. Corroborating these findings, Larson et al. (2022) affirmed that incorporating multiple representations enhances students' grasp of mathematical concepts, with (Van Dooren, et al., 2020) concurring that visual representations notably enhance students' capacity to tackle intricate mathematical challenges.

Conversely, the adoption of alternative teaching tools like manipulatives, graphics, and diverse representations for linear equations remains less widespread (Larbi, & Okyere, 2014), reflecting a pattern similar to (Delice, & Sevimli, 2010) where teachers' utilization of multiple representations fell short of expectations. Similarly, (Gagatsis & Shiakalli, 2004) observed minimal incorporation of graphic representation in teaching. This may be attributed to teachers grappling with the integration of diverse representations within their instructional milieu which is highlighted by (Birgin et al., 2012) as challenges elucidated in the current study. These challenges are mirrored in teachers' accounts of reasons for underutilization, such as a dearth of innovative ideas (Beatty, 2010).

The findings also indicated a prevalent preference among teachers for utilizing algebraic representation when teaching linear equations. This inclination stemmed from various factors, including its simplicity, ease of use, efficiency, widespread familiarity, and comprehensibility (Celik, & Baki, 2007). These considerations played a pivotal role in shaping teachers' selection of algebraic representation as their preferred instructional approach. This discovery aligns with previous research by (Bal, 2014), wherein educators justified their utilization of algebraic representation by emphasizing its clarity. This phenomenon can potentially be attributed to teachers' perceptions that their students grasp linear equations more readily when presented using algebraic representation. Furthermore, (Boaler, 2016) proposed that incorporating diverse representations into instruction could cultivate heightened student engagement and interest in mathematics. By employing this approach, educators can establish a dynamic and interactive classroom atmosphere that caters to an array of learning preferences and styles.

Furthermore, the study found that teachers were less inclined to employ manipulatives, graphics, and multiple representations for teaching linear equations. This reluctance emanated from reasons such as time constraints, perceived difficulty for students, scarcity of materials, lack of innovative ideas, and potential student confusion (Huntley et al., 2007). This finding corresponds with the outcomes of the research conducted by Bal (2014), reinforcing the notion that teachers' adoption of various representations hinges on their grasp of the concepts. When teachers lack a robust understanding of a particular representation, the likelihood of its incorporation into their teaching practices diminishes. Thus, it is unsurprising that only a minority of teachers opted to integrate manipulatives, graphics, and multiple representations into their linear equation instruction.

Moreover, the analysis of students' performance on LEAT, as assessed through ANCOVA, unveiled a significant and noteworthy distinction among the various instructional groups, with the advantage of favouring those exposed to instructions utilizing multiple representations. Notably, the adjusted mean differences among these groups also displayed statistical significance. Among the groups, Experimental Group 1 exhibited the highest adjusted mean, signifying superior performance. This outcome is potentially attributable to the fact that students in Experimental Group 1 had the opportunity to engage with linear equations through a triad of representations: manipulatives, graphics, and algebraic (DeJarnette et al., 2020).

Similarly, students in Experimental Group 2, who encountered linear equations via the combined use of manipulatives and algebraic representation, displayed commendable performance, reflected in the second-highest adjusted mean. On the contrary, the Control group demonstrated the lowest adjusted mean, indicating comparatively weaker performance. The Control group lacked a profound comprehension of this mathematical concept, contrasting with the Experimental groups, who benefitted from instruction employing diverse representations. This aligns with the findings of (Bittinger et al., 2013). The adoption of multiple representations not only facilitated a more robust conceptual grasp of function but also contributed to heightened student engagement with mathematical concepts and the establishment of meaningful connections between distinct mathematical ideas, as proposed in (Llinares et al., 2021).

These findings align with the research conducted by Doktoroglu (2013), which delved into the impact of utilizing dynamic mathematics software to teach linear equations to seventh-grade students. The outcomes revealed a noteworthy enhancement in students' proficiency with linear equations for the Experimental group that engaged with various representations offered by the dynamic mathematics software. Conversely, no substantial effects were observed in activities that could not leverage the diverse representations provided by the data. Consequently, these results harmonize with the conclusions drawn by Bal (2014) and (Cikla, 2004). This suggests a potential correlation between employing multiple representations in mathematical instruction and a potential increase in students' academic accomplishments.

6. Limitation

This paper has two limitations that have to be considered. First, The use of a questionnaire to collect data may have limitations. The questionnaire, for example, may not capture key facets of the research problem that necessitate more in-depth qualitative data collection approaches. Second, the study occurred in three different schools with three different teachers. Since every school has a different climate and each of the teachers may use different teaching methods, the results may be different if the study were to be conducted in the same school under one teacher.

7. Recommendation

Drawing from the insights highlighted earlier, the following recommendations are required to guide policy formulation and decision-making:

- Multiple representations support the abstraction of mathematical concepts and enhance students' learning. Therefore, teaching linear equations in one variable should not be limited to one representation such as algebraic. Other representations such as manipulatives and graphics should be used alongside the traditional use of algebraic representation.

- Teachers need to acknowledge the unique learning styles and preferences of their students, tailoring instructional approaches by employing a diverse collection of representations. This may encompass incorporating visual aids, hands-on activities, real-world applications, and technology-based tools to align with the distinct needs of each learner.
- Parents can actively contribute by fostering a supportive learning environment within the home. Encouraging their children to engage with various representations and being readily available to address queries and provide guidance helps fortify the advantages of the educational guidance received in the classroom.
- Students are encouraged to cultivate proficiency in integrating multiple representations when tackling problems related to linear equations. This involves tasks such as generating graphs based on provided equations, interpreting data from tables, and fluidly transitioning between different representations to enhance problem-solving efficacy.
- The Ghana Education Service need to introduce professional development courses for mathematics teachers at the basic level to update teachers' knowledge and skills concerning the use of various representations in teaching linear equations in one variable. This will make it possible to integrate unfamiliar representations easily in the course of teaching and learning linear equations in one variable.
- Mathematics syllabi and textbooks should be designed to include all representations necessary in teaching linear equations in one variable. This will make the teaching and learning materials useful and relevant to teachers and students.
- The Ghana Education Service should intensify school supervision to ensure that teachers apply appropriate teaching methodologies to achieve learning outcomes. This will help the students to appreciate what is being taught in the classroom.
- Representational materials such as computers, graphs, algebra tiles and other related accessories should be made available to mathematics teachers for them to use in teaching linear equations in one variable.
- Future researchers are required to emphasize equipping educators with ample resources and designing robust training programs to enable them to adeptly incorporate multiple representations-based instruction for teaching linear equations.
- Future research needs to engage students in the creation and assessment of instructional materials rooted in multiple representations. Soliciting feedback from students regarding their learning experiences and preferences can guide the ongoing refinement of teaching strategies, ensuring they are tailored to be more student-focused and effective.

8. Conclusion

The researchers illustrated the utilization of diverse instructional approaches in teaching linear equations to eighth-grade students. The investigation centred around the selection of teaching methods by educators when instructing linear equations with a single variable. It explored the factors influencing educators' preference for specific teaching methods and examined the impact of incorporating multiple representations in instruction on students' performance in linear equations.

The study discovered that a significant proportion of teachers favoured employing algebraic representation due to its perceived advantages such as simplicity, speed, familiarity, widespread usage, and comprehensibility. In contrast, a limited number of teachers opted for manipulatives, visual aids, and multiple representations, primarily due to concerns related to time constraints, perceived complexity for students, scarcity of resources, creative limitations, curricular constraints, and perceived lack of relevance. Moreover, the investigation revealed noteworthy enhancements in students' performance on linear equations assessments after the implementation of instruction grounded in multiple representations. Therefore, it can be inferred that incorporating diverse representations in the teaching of linear equations is beneficial for enhancing students' comprehension and achievement in the subject.

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